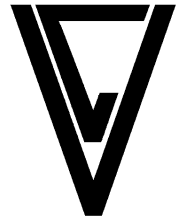




Universidad Nacional Autónoma de México
Facultad de Ingeniería
División de Ciencias Básicas
Coordinación de Matemáticas
CÁLCULO VECTORIAL



SEGUNDO EXAMEN EXTRAORDINARIO

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Semestre: 2017-1

Duración máxima: 2 horas

Nombre: _____ No. de cuenta: _____

1. Determinar la naturaleza de los puntos críticos de la función

$$f(x, y) = 3x^4 + 4x^3 - 30x^2 + 6y^2 - 12xy$$

15 PUNTOS

2. Calcular el ángulo de intersección entre la curva C y la superficie S representadas por:

$$C : \vec{r}_1(t) = (2\cos t)i + (2\sin t)j + (t)k ; t \in [0, \pi]$$

y

$$S : \begin{cases} x = 2(\sin u) \cos v \\ y = 2(\sin u) \sin v \\ z = 2 \cos u \end{cases} \quad u \in [0, \pi] ; v \in [0, 2\pi]$$

en el punto donde $|\vec{r}_1(t)| = 2$

15 PUNTOS

3. Sea el campo vectorial:

$$\vec{F}(r, \theta, z) = (\sin^2 \theta) \hat{e}_r + (\sin 2\theta) \hat{e}_\theta + \hat{e}_z$$

Determinar si \vec{F} es un campo conservativo, si lo es, obtener su función potencial.

15 PUNTOS

4. Sea la transformación:

$$T: \begin{cases} u = 2x^2 - y^2 + 5 \\ v = 2xy^2 + 4 \end{cases}$$

Determinar:

- Si el sistema de coordenadas (u, v) es ortogonal.
- Los factores de escala h_u y h_v
- Los vectores base \hat{e}_u y \hat{e}_v
- El jacobiano $J\left(\frac{x, y}{u, v}\right)$

20 PUNTOS

5. Obtener la integral:

$$I = \int_C (x + y + z) dx + x^2 y z dy + 3x y z^2 dz \quad \text{desde el punto } A(0,0,1)$$

hasta el punto $B(1,1,1)$.

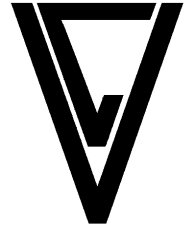
Donde C es el segmento de parábola $C: \begin{cases} y = x^2 \\ z = 1 \end{cases}$

15 PUNTOS

6. Calcular la circulación total del campo vectorial $\vec{F}(x, y, z) = yi - xj + zk$ alrededor de la curva representada por:

$$C: \begin{cases} x^2 + y^2 + z^2 = 8 \\ x^2 + y^2 = z^2 \end{cases} ; \quad z \geq 0$$

20 PUNTOS



Semestre: 2017-1

1. $f(x, y) = 3x^4 + 4x^3 - 30x^2 + 6y^2 - 12xy$

$$\frac{\partial f}{\partial x} = 12x^3 + 12x^2 - 60x - 12y = 0$$

$$\frac{\partial f}{\partial y} = 12y - 12x = 0$$

$$y = x$$

$$12x^3 + 12x^2 - 60x - 12y = 0$$

$$12(x^3 + x^2 - 6x) = 0$$

$$x(x^2 + x - 6) = 0$$

$$x(x+3)(x-2) = 0$$

Puntos críticos $P_1(0,0)$, $P_2(-3,-3)$, $P_3(2,2)$

$$H = \begin{vmatrix} 36x^2 + 24x - 60 & -12 \\ -12 & 12 \end{vmatrix} = 12^2 \begin{vmatrix} 3x^2 + 2x - 5 & -1 \\ -1 & 1 \end{vmatrix}$$

Para $P_1(0,0)$; $H|_{P_1} = 12^2 \begin{vmatrix} -5 & -1 \\ -1 & 1 \end{vmatrix} = -12^2(6) < 0 \therefore P_1$ es un P.S.

Para $P_2(-3,-3)$; $H|_{P_2} = 12^2 \begin{vmatrix} 16 & -1 \\ -1 & 1 \end{vmatrix} = 12^2(15) > 0$ $\frac{\partial^2 f}{\partial x^2}|_{P_2} = 16 > 0 \therefore P_2$ existe un mínimo

Para $P_3(2,2)$; $H|_{P_3} = 12^2 \begin{vmatrix} 11 & -1 \\ -1 & 1 \end{vmatrix} = 12^2(10) > 0$; $\frac{\partial^2 f}{\partial x^2}|_{P_3} = 11 > 0 \therefore P_3$ existe un mínimo

2.

$$\bar{r}_1(t) = 2 \cos t \, i + 2 \sin t \, j + t \, k$$

$$\bar{r}_2(u, v) = 2 \sin u \cos v \, i + 2 \sin u \sin v \, j + 2 \cos u \, k$$

$$\cos t = 2 \sin u \cos v$$

$$\sin t = 2 \sin u \sin v$$

$$t = 2 \cos u$$

$$\text{si } t = 0 \quad \cos u = 0 \rightarrow u = \frac{\pi}{2}$$

$$\frac{\sin t}{\cos t} = \tan t = \tan v \rightarrow v = 0$$

$$\frac{\partial \bar{r}_2}{\partial u} = 2 \cos u \cos v \, i + 2 \cos u \sin v \, j - 2 \sin u \, k$$

$$\frac{\partial \bar{r}_2}{\partial v} = -2 \sin u \sin v \, i + 2 \sin u \cos v \, j$$

$$\bar{N}_2 = \frac{\partial \bar{r}_2}{\partial u} \times \frac{\partial \bar{r}_2}{\partial v} = \begin{vmatrix} i & j & k \\ 0 & 0 & -2 \\ 0 & -2 & 0 \end{vmatrix} = -4i$$

$$\frac{d\bar{r}_1}{dt} = -2 \sin t \, i + 2 \cos t \, j + k \quad \frac{d\bar{r}_1}{dt} \Big|_P = 2j + k$$

$$\sin \alpha = \frac{(-4, 0, 0) \cdot (0, 2, 1)}{(4)(\sqrt{5})} = 0 \rightarrow \alpha = 0^\circ$$

$$\bar{r}_1(t) = \sqrt{4 \cos^2 t + 4 \sin^2 t + t^2}$$

$$= \sqrt{4 + t^2}$$

$$\sqrt{4 + t^2} = 2$$

$$4 + t^2 = 4 \rightarrow t^2 = 0 \quad t = 0$$

$$\frac{\partial \bar{r}_2}{\partial u} \Big|_P = 0i + 0j - 2k$$

$$\frac{\partial \bar{r}_2}{\partial v} \Big|_P = 0i - 2j$$

15 PUNTOS

3.

$$\bar{F}(p, \theta, z) = \sin^2 \theta \, e_p + \sin 2\theta \, e_\theta + e_z$$

$$\nabla_x \bar{F} = \frac{1}{p} \begin{vmatrix} \hat{e}_p & p \hat{e}_\theta & \hat{e}_z \\ \frac{\partial}{\partial p} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ \sin^2 \theta & p \sin 2\theta & 1 \end{vmatrix} = 0 \hat{e}_p - 0 \hat{e}_\theta + (\sin 2\theta - 2 \sin \theta \cos \theta) e_z$$

$$\text{como } \sin 2\theta = 2 \sin \theta \cos \theta ;$$

$$\sin 2\theta - 2 \sin \theta \cos \theta = 0$$

$\therefore \bar{F}$ es conservativo

$$\text{como } \bar{F} = \nabla f \rightarrow \frac{\partial f}{\partial p} = \sin^2 \theta ;$$

$$\frac{\partial f}{\partial \theta} = p \sin 2\theta ; \quad \frac{\partial f}{\partial z} = 1$$

$$= 2 p \sin \theta \cos \theta$$

$$\text{Integrando } f = p \sin^2 \theta + z + c$$

15 PUNTOS

4.

$$T : \begin{cases} u = 2x^2 - y^2 + 5 \\ v = 2xy^2 + 4 \end{cases}$$

$$\bar{\nabla}u = 4xi - 2yj$$

$$\bar{\nabla}v = 2y^2i + 4xyj$$

$$\bar{\nabla}u \cdot \bar{\nabla}v = 8xy^2 - 8xy^2 = 0$$

\therefore el sistema es ortogonal

$$h_u = \frac{1}{|\bar{\nabla}u|} = \frac{1}{\sqrt{16x^2 + 4y^2}} = \frac{1}{2\sqrt{4x^2 + y^2}}$$

$$h_v = \frac{1}{|\bar{\nabla}v|} = \frac{1}{\sqrt{4y^4 + 16x^2y^2}} = \frac{1}{2y\sqrt{y^2 + 4x^2}}$$

$$e_u = \frac{2xi - yj}{\sqrt{4x^2 + y^2}} \quad e_v = \frac{yi + 2xj}{\sqrt{4x^2 + y^2}}$$

$$\begin{aligned} J\left(\frac{x, y}{u, v}\right) &= h_u h_v = \frac{1}{\sqrt{16x^2 + 4y^2}} \frac{1}{\sqrt{4y^4 + 16x^2y^2}} \\ &= \frac{1}{2\sqrt{4x^2 + y^2}} \frac{1}{2y\sqrt{y^2 + 4x^2}} \\ &= \frac{1}{4y\sqrt{4x^2 + y^2}} \end{aligned}$$

20 PUNTOS

5.

$$\int_C (x, y, z)dx + x^2 yzdy + (3xyz^2)dz =$$

$$\int_0^1 (x + x^2 + 1)dx + x^2(x^2)(1)(2x)dx + 3(x)(x^2)(1)^2(0)dx$$

$$\int_0^1 (x + x^2 + 1 + 2x^5)dx = \frac{2x^6}{6} + \frac{x^3}{3} + \frac{x^2}{2} + x \Big|_0^1$$

$$= \frac{1}{3} + \frac{1}{3} + \frac{1}{2} + 1 = \frac{2 + 2 + 3 + 6}{6} = \frac{13}{6}$$

15 PUNTOS

6.

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_R \vec{\nabla} \times \vec{r} \cdot \vec{n} \, dS$$

$$x^2 + y^2 + z^2 = 8$$

$$\text{restando } z^2 = 8 - z^2$$

$$x^2 + y^2 = z^2$$

$$z^2 = 4$$

$$z = \pm 2$$

$$z = 2 ; \quad x^2 + y^2 = 4$$

Parametrizando

$$x = 2 \cos t$$

$$t : 0 \rightarrow 2\pi$$

$$y = 2 \sin t$$

$$z = 2$$

$$\oint_C \vec{F} \cdot d\vec{r} = \int_0^{2\pi} 2 \sin t (-2 \sin t) - (2 \cos t)(2 \cos t) + 2(0) \, dx$$

$$= -4 \int_0^{2\pi} \sin^2 t + \cos^2 t \, dt$$

$$= -4 \int_0^{2\pi} dt = -4t \Big|_0^{2\pi} = -8\pi \text{ u de circulación}$$