Universidad Nacional Autónoma de México
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División de Ciencias Básicas
Coordinación de Matemáticas
Cálculo Vectorial
Segundo Examen Extraordinario Solución

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1.

$$S: 16x^2 + 4y^2 + 9z^2 = 144; \qquad V = xyz$$

$$F = xyz + \lambda(16x^2 + 4y^2 + 9z^2 - 144)$$

$$\frac{\partial F}{\partial x} = yz + 32x\lambda = 0 \quad \Rightarrow \quad yz = -32x\lambda$$
$$\frac{\partial F}{\partial y} = xz + 8y\lambda = 0 \quad \Rightarrow \quad xz = -8y\lambda$$
$$\frac{\partial F}{\partial z} = xy + 18z\lambda = 0 \quad \Rightarrow \quad xy = -18z\lambda$$

$$\lambda = \frac{yz}{-32x}; \qquad \lambda = \frac{xz}{-8y}; \qquad \lambda = \frac{xy}{-18z}$$

$$\frac{yz}{-32x} = \frac{xz}{-8y} \qquad\qquad \frac{xz}{-8y} = \frac{xy}{-18z}$$

$$x = \pm y\left(\frac{1}{2}\right) \qquad\qquad z = \pm\frac{2}{3}y$$

$$16\left(\frac{y^2}{4}\right) + 4y^2 + 9\left(\frac{4}{9}y^2\right) = 144$$

$$y = 2\sqrt{3}; \qquad x = \sqrt{3}; \qquad z = \frac{4}{3}\sqrt{3}$$

$$l = 2x = 2\sqrt{3}$$
$$a = 2y = 4\sqrt{3}$$
$$h = \frac{8}{3}\sqrt{3} = 2z$$

2.

$$C:\begin{cases} y = x^2 + y^2 \\ x + z = 0 \end{cases} \qquad P(1,\ 2, -1)$$

Es una curva contenida en un plano, esto es una curva plana

$\therefore \tau = 0 \quad y \quad x + z = 0 \quad es \ el \ plano \ osculador$

$$K = \frac{\left|\overline{r}\,'x\,\overline{r}\,''\right|}{\left|\overline{r}\,'\right|^3}$$

$C : \overline{r}(t)$

$si \quad x = t, \quad z = -t \quad y \quad y = 2t^2$

$\overline{r}(t) = (t, 2t^2, -t) \quad por \quad t = 1 \quad se \ obtiene \quad P.$

$\overline{r}\,'(t)\big|_{t=1} = (1, 4, -1)\ ; \qquad \overline{r}\,''(t)\big|_{t=1} = (0, 4, 0)$

$\overline{r}\,'x\ \overline{r}\,'' = 4i + 4k \ \Rightarrow \ \left|\overline{r}\,'x\,\overline{r}\,''\right| = 4\sqrt{2}$

$\left|\overline{r}\,'\right|^3 = 54\sqrt{2}$

$$\therefore K = \frac{2}{27}$$

3.

$$\overline{F}\,(r, \theta) = 3r^2 sen\,\theta\,\hat{e}_r + r^2 \cos\theta\, e_\theta$$

$$\frac{\partial P}{\partial \theta} =^{¿?} r\,\frac{\partial \theta}{\partial r}$$

$3r^2 \cos\ \theta = 3r^2 \cos\ \theta \quad \therefore\ es\ conservativo$

$I_1 = \int 3r^2 sen\,\theta\, dr \ = \ r^3\ sen\,\theta + c_1$

$I_2 = \int r^3 \cos\theta\, d\theta \ = r^3 sen\,\theta + c_2$

$\phi = I_1 \cup I_2 = r^3\ sen\,\theta + c$

4.

$$I=\int_{A^C}^{B} xy^2 ds \qquad C: x^2+y^2 \qquad A(1,0)\quad B(0,1)$$

$$\therefore x=\cos\theta \qquad \theta\,\varepsilon\left[0,\frac{\pi}{2}\right]$$

$$y=sen\,\theta$$

$$I=\int_0^{\frac{\pi}{2}} sen^2\theta\cos\theta\sqrt{\cos^2\theta+sen^2\theta}\;d\theta$$

$$=\int_0^{\frac{\pi}{2}} sen^2\theta\cos\theta\;d\theta=\frac{1}{3}sen^3\,\theta\Big|_0^{\frac{\pi}{2}}=\frac{1}{3}$$

5.

$$I=\iint_R e^{-x^2-y^2}dxdy$$

$$I=\int_0^{\frac{\pi}{2}}\int_1^2 e^{-r^2}r\,dr\,d\theta=\int_0^{\frac{\pi}{2}}-\frac{1}{2}\;e^{-r^2}\Big|_1^2 d\theta=-\frac{1}{2}(e^{-4}-e^{-1})\Big|_0^{\frac{\pi}{2}} d\theta$$

$$I=-\frac{1}{4}\pi(e^{-4}-e^{-1})\Big|$$

6.

$$\overline{F}=(x,y,z)$$

$$S:\begin{cases} z=\sqrt{x^2+y^2} \\ z=4\end{cases} \qquad cono$$

$$I=\iiint_D div\overline{F}dV$$

$$div\overline{F}=\overline{\nabla}\bullet\overline{F}=3$$

$$I=3\iiint_D dV=3V$$

$$I=3\frac{1}{3}\pi(4)^2 4$$

$$I=64\pi \qquad v.\ de\ \ flujo$$