Sistemas de múltiple grado de libertad en vibraciones.

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- Ecuación diferencial de un modelo con resorte y amortiguador.
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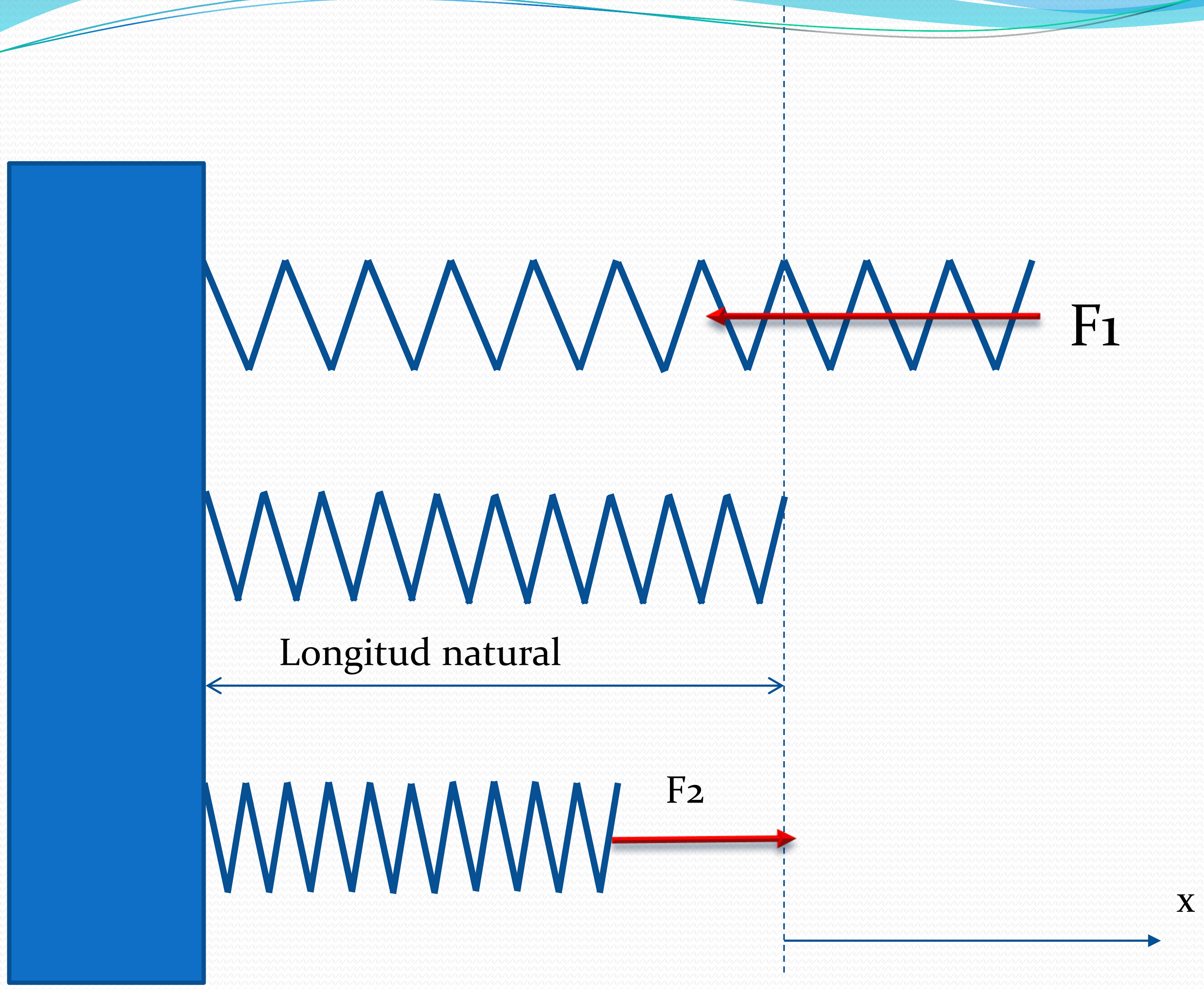
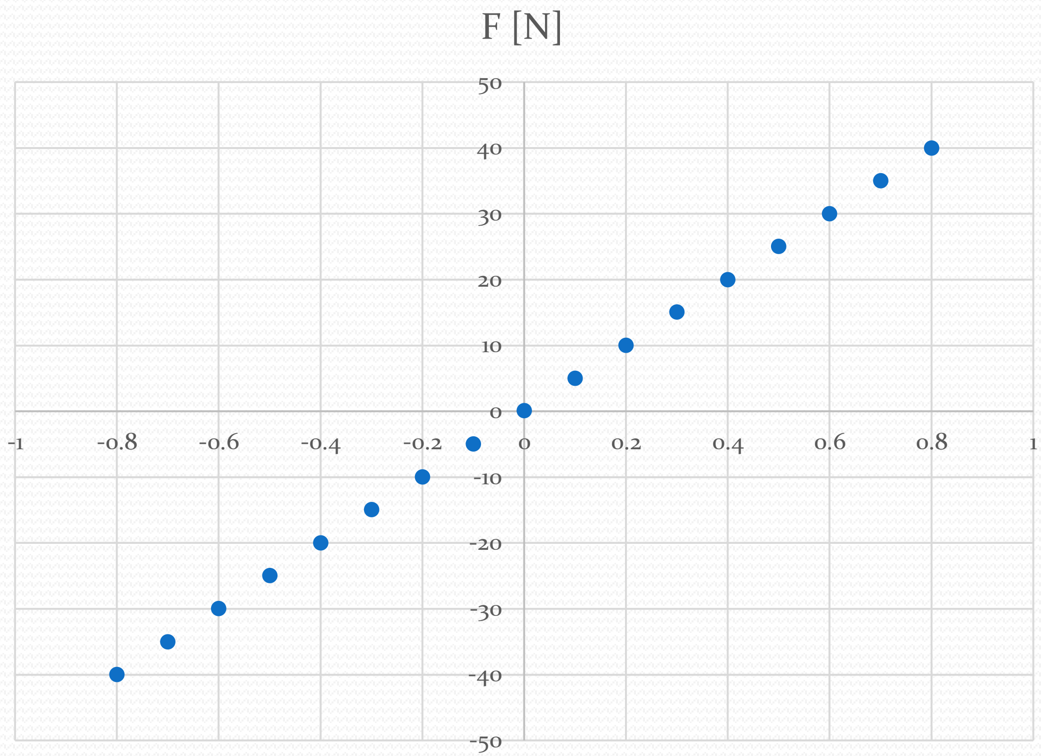
- Sistemas de múltiple grado de libertad.

Ley de Hook.


F1
Longitud natural
F2
x

x [m]	F [N]
0	0.08520243
0.1	5.02510807
0.2	10.0472738
0.3	15.0850686
0.4	20.0030134
0.5	25.0426012
0.6	30.0113097
0.7	35.0270566
0.8	40.0071304
-0.1	-4.9579192
-0.2	-9.94631297
-0.3	-14.9351541
-0.4	-19.995591
-0.5	-24.9546295
-0.6	-29.9433512
-0.7	-34.9965123
-0.8	-39.9722169



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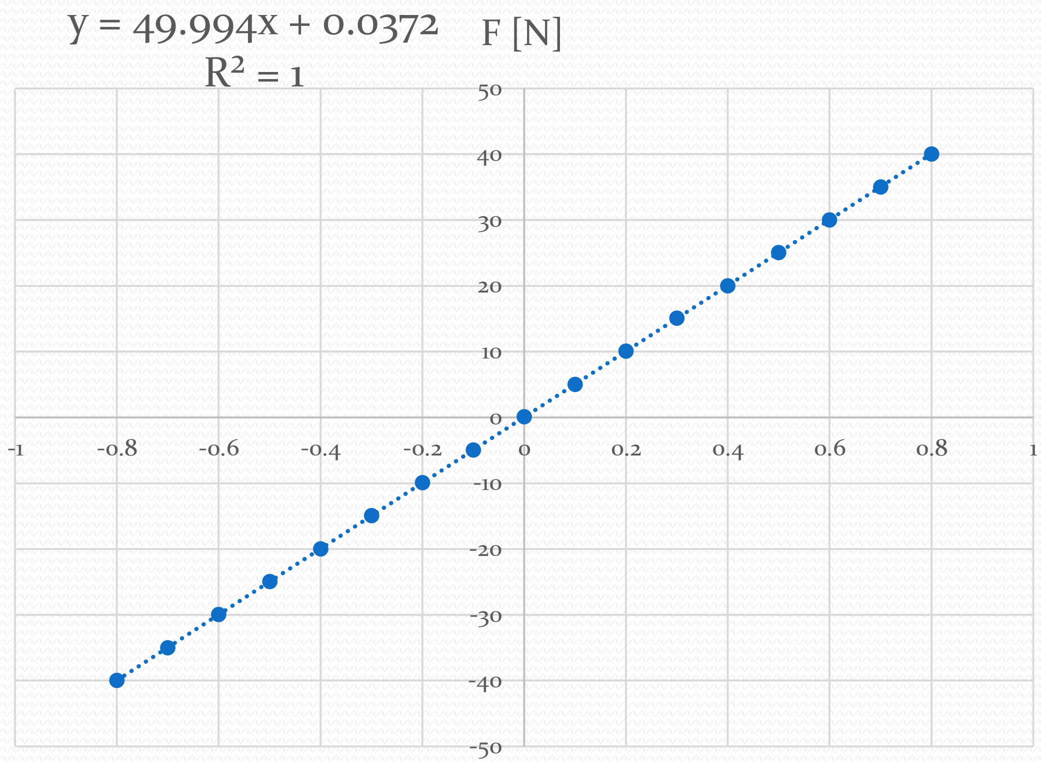

x [m]	F [N]
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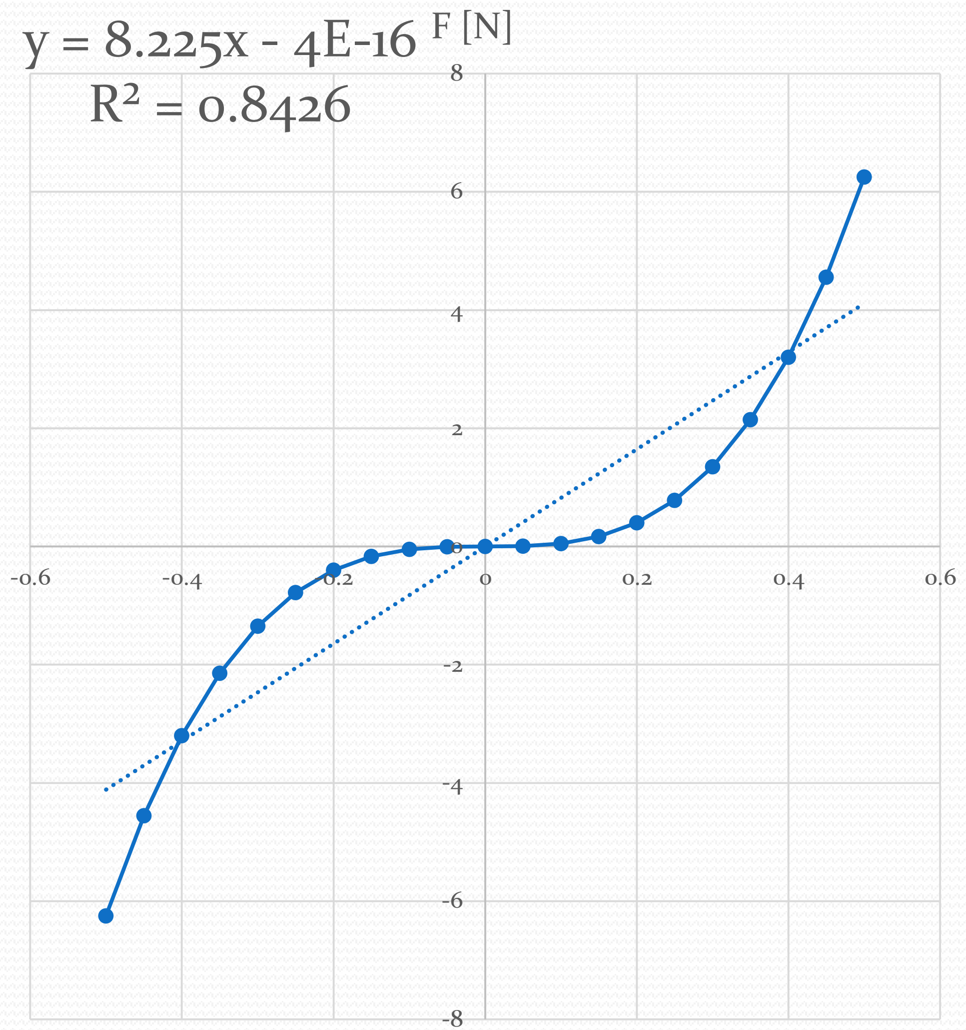
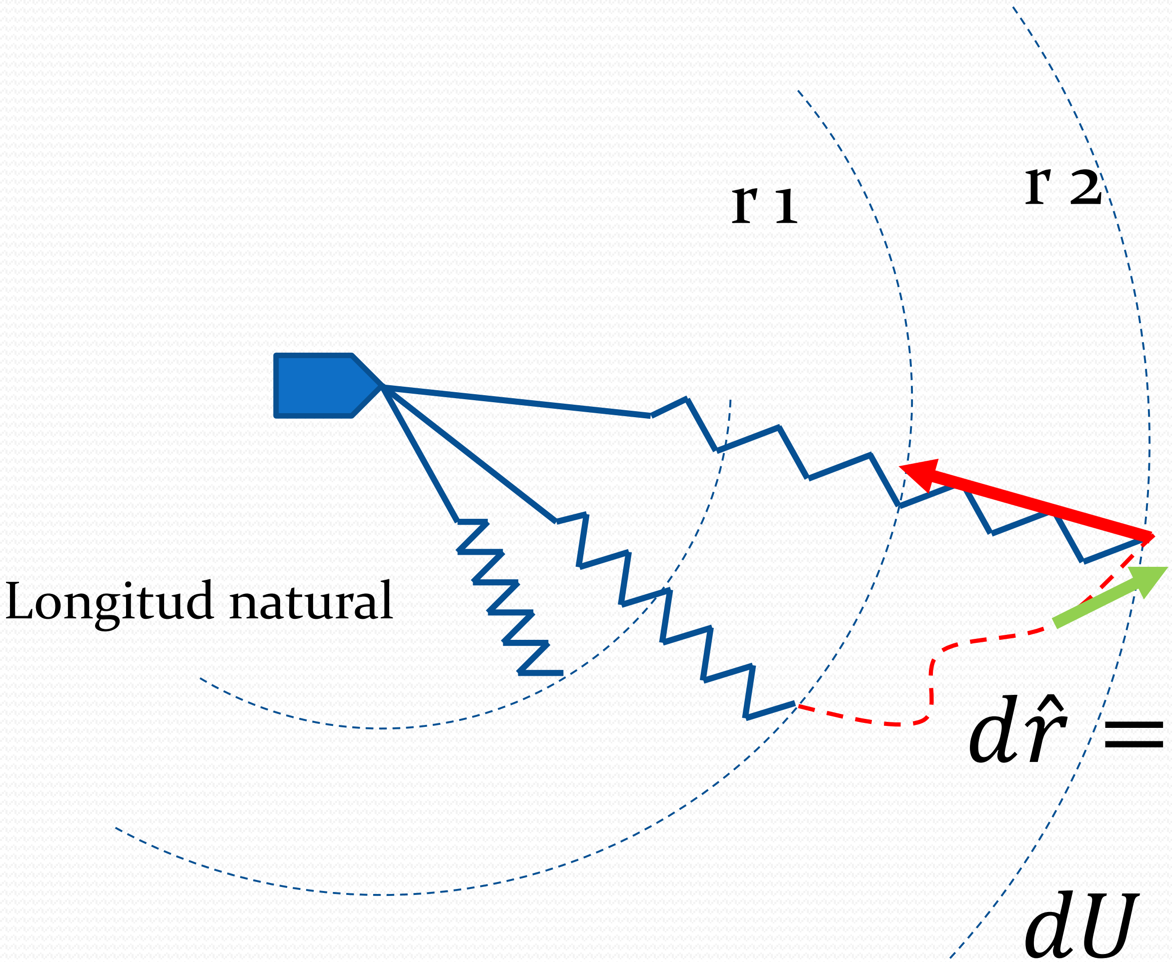
0.8	40.0071304
-0.1	-4.9579192
-0.2	-9.94631297
-0.3	-14.9351541
-0.4	-19.995591
-0.5	-24.9546295
-0.6	-29.9433512
-0.7	-34.9965123
-0.8	-39.9722169



Ley de Hook

La fuerza que ejerce un resorte lineal es proporcional a la deformación que tiene a partir de su longitud natural y en dirección contraria a la deformación.

$$\overrightarrow{F_k} = -\,k\,x\,\hat{\imath}$$



$$\vec{F} = -k\, r\, \widehat{er}$$

$$d\hat{r} = dr\, \widehat{er} + r d\theta\, \widehat{e\theta}$$

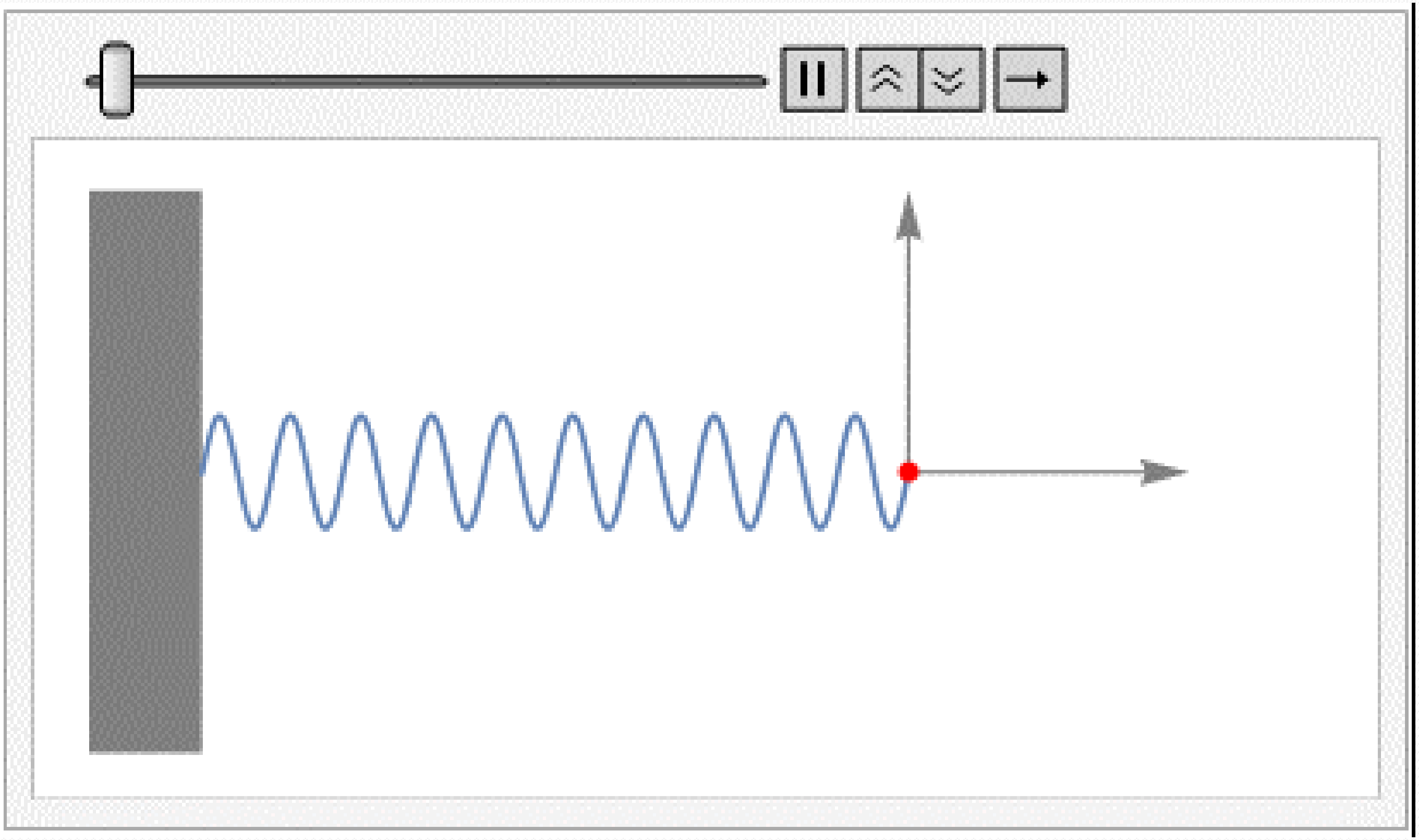
$$dU = -k\, r\, dr$$

$$dU = -kr\ dr$$

$$\text{U12} = \int_1^2 -k\ r\ dr = -\frac{k\ {r_2}^2}{2} + \frac{k\ {r_1}^2}{2}$$

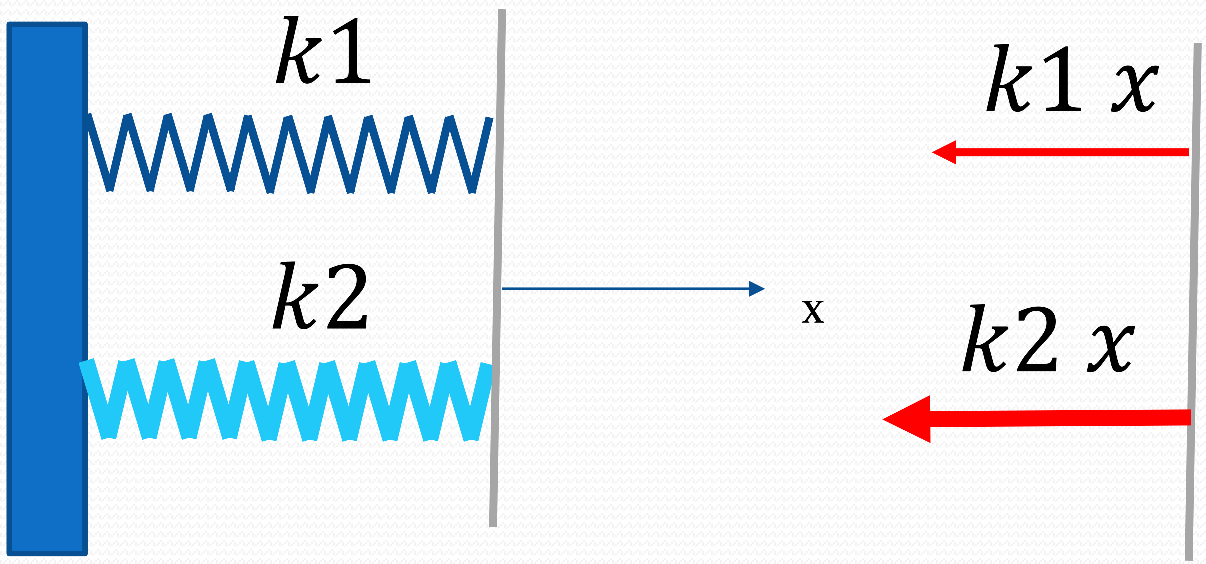
$$V_k = \frac{k\ r^2}{2}$$

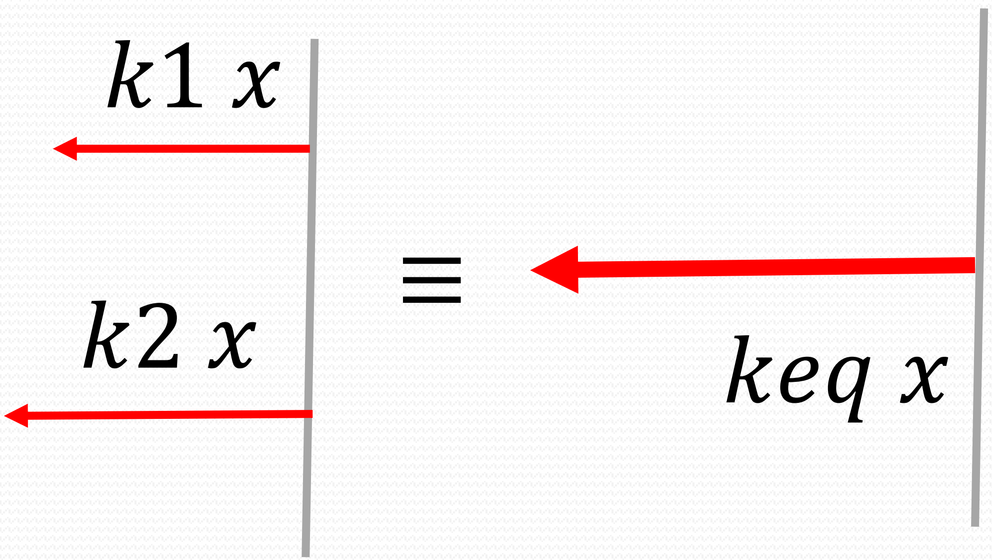
$$V_{k2} + V_{k1} = \frac{k\ {r_2}^2}{2} - \frac{k\ {r_1}^2}{2}$$



Resortes en serie, resortes en paralelo.

Resortes en paralelo





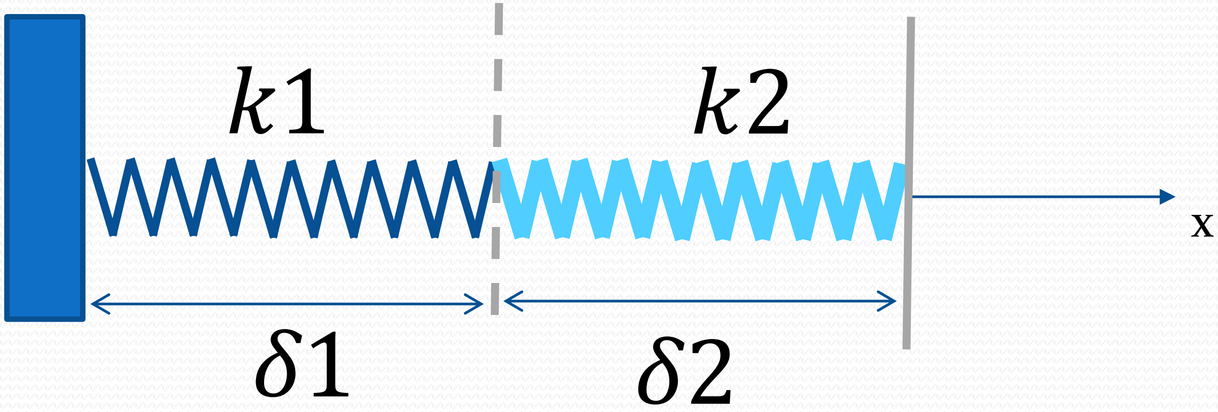
$$k1\,x + k2\,x = \sum F$$

$$keqx = \sum F$$

$$k1\,x + k2\,x = keq\,x$$

$$k1 + k2 = keq$$

Resortes en serie



$$F = k1\,\delta 1 = k2\,\delta 2 = keq\;x$$

$$F = k1\,\delta 1 = k2\,\delta 2 = keq\,x$$

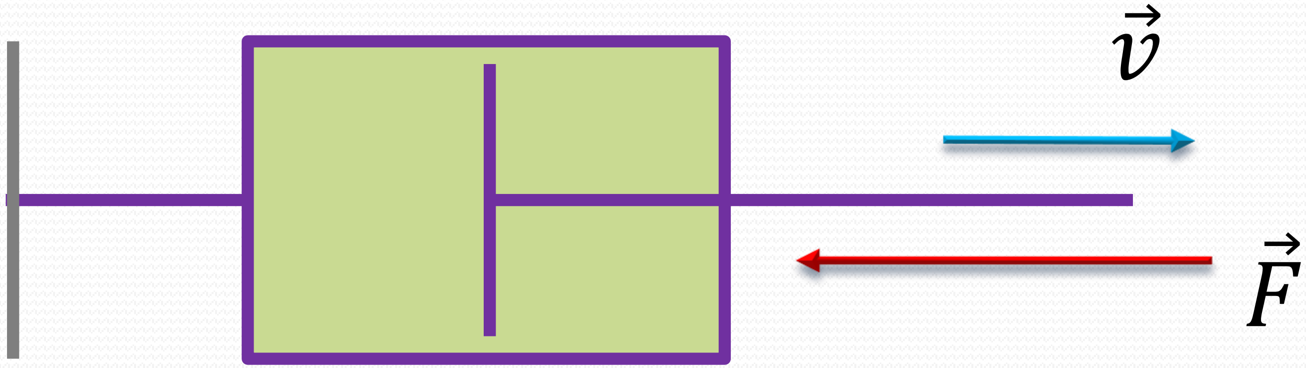
$$\delta 1 + \delta 2 = x$$

$$\frac{F}{k1} + \frac{F}{k2} = \frac{F}{keq}$$

$$keq = \frac{1}{\sum \frac{1}{k_i}}$$

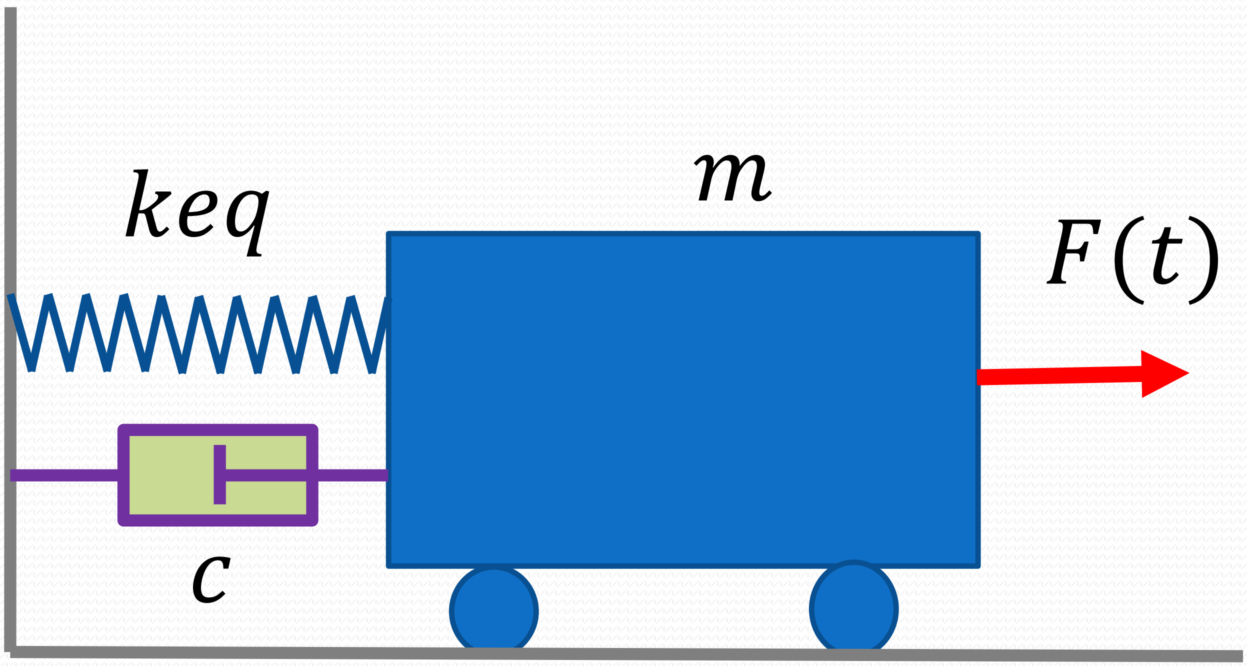
Ecuación diferencial de un modelo con resorte y amortiguador.

Amortiguador viscoso

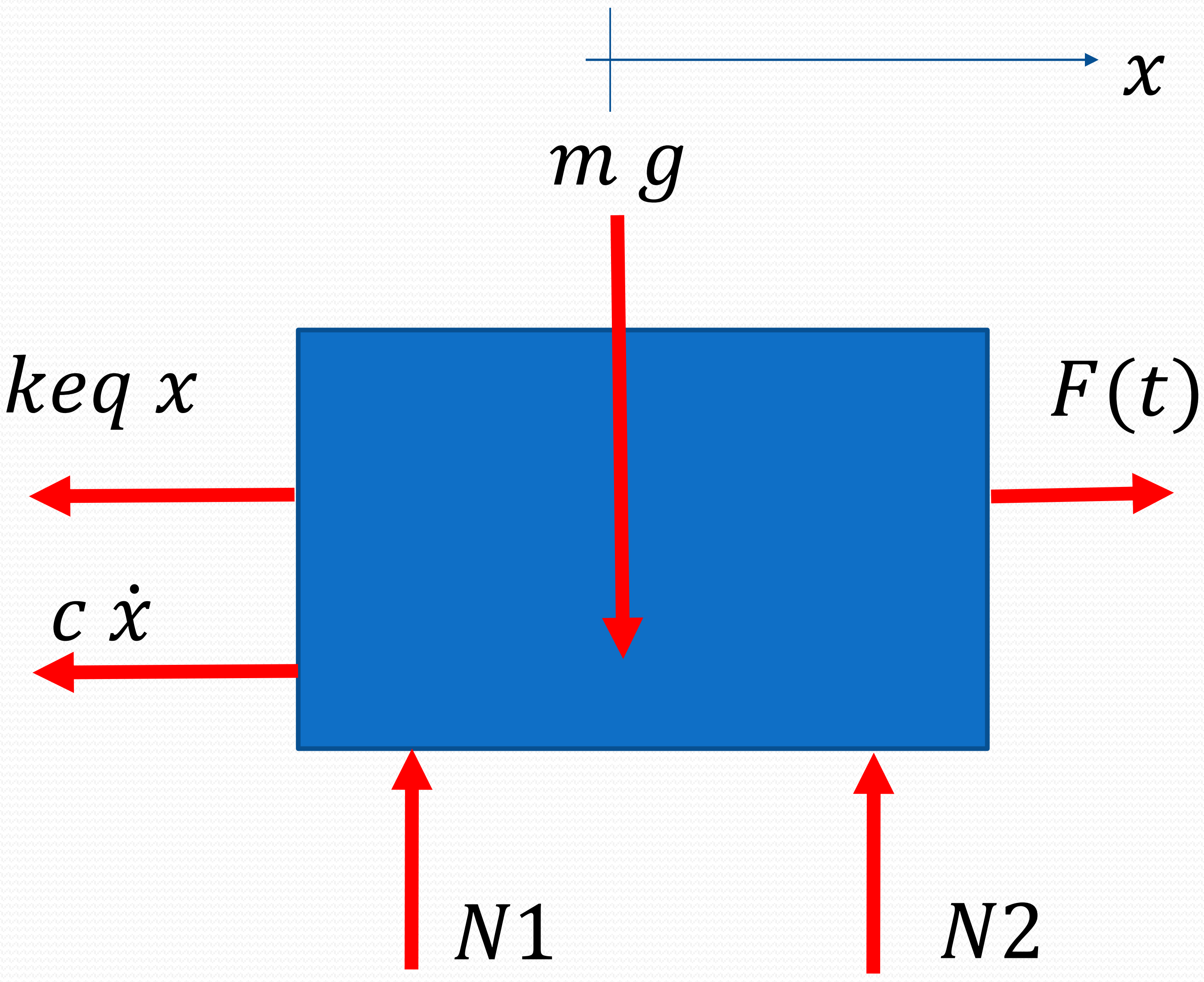


La fuerza que ejerce un amortiguador viscoso es proporcional a la velocidad de deformación entre sus extremos relativos y en dirección contraria a dicha velocidad.

$$\overrightarrow{F_c} = -c\,\dot{x}\,\hat{\imath}$$

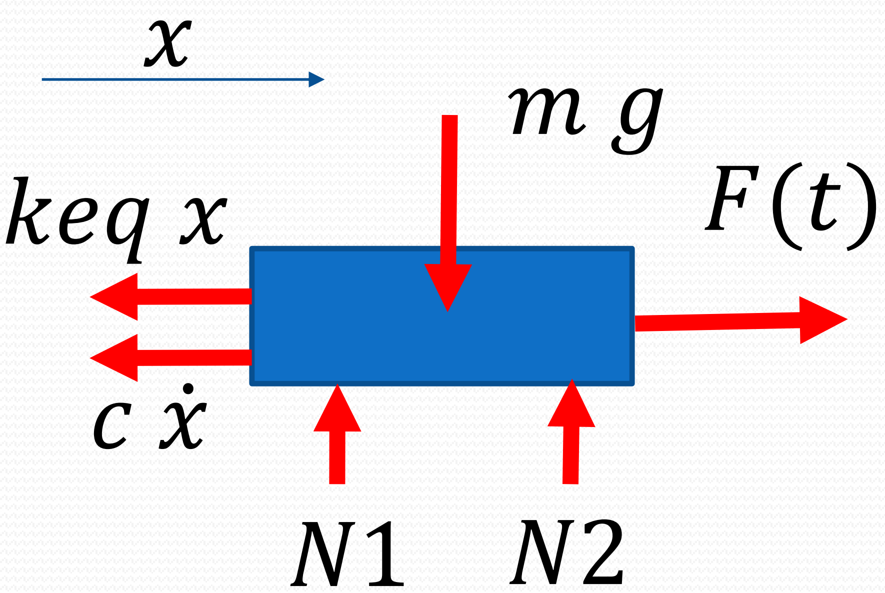


keq
m
F(t)
c


x
$m\ g$
$keq\ x$
$F(t)$
$c\ \dot{x}$
$N1$
$N2$

$$\sum Fy = -m\ g + N1 + N2 = 0$$

$$m\ g = N1 + N2$$



$$\sum Fx = -kx - c\dot{x} + F(t) = m\ddot{x}$$

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\left(D^2 + D\frac{c}{m} + \frac{k}{m}\right)x = 0$$

$$\lambda^2 + \lambda\frac{c}{m} + \frac{k}{m} = 0$$

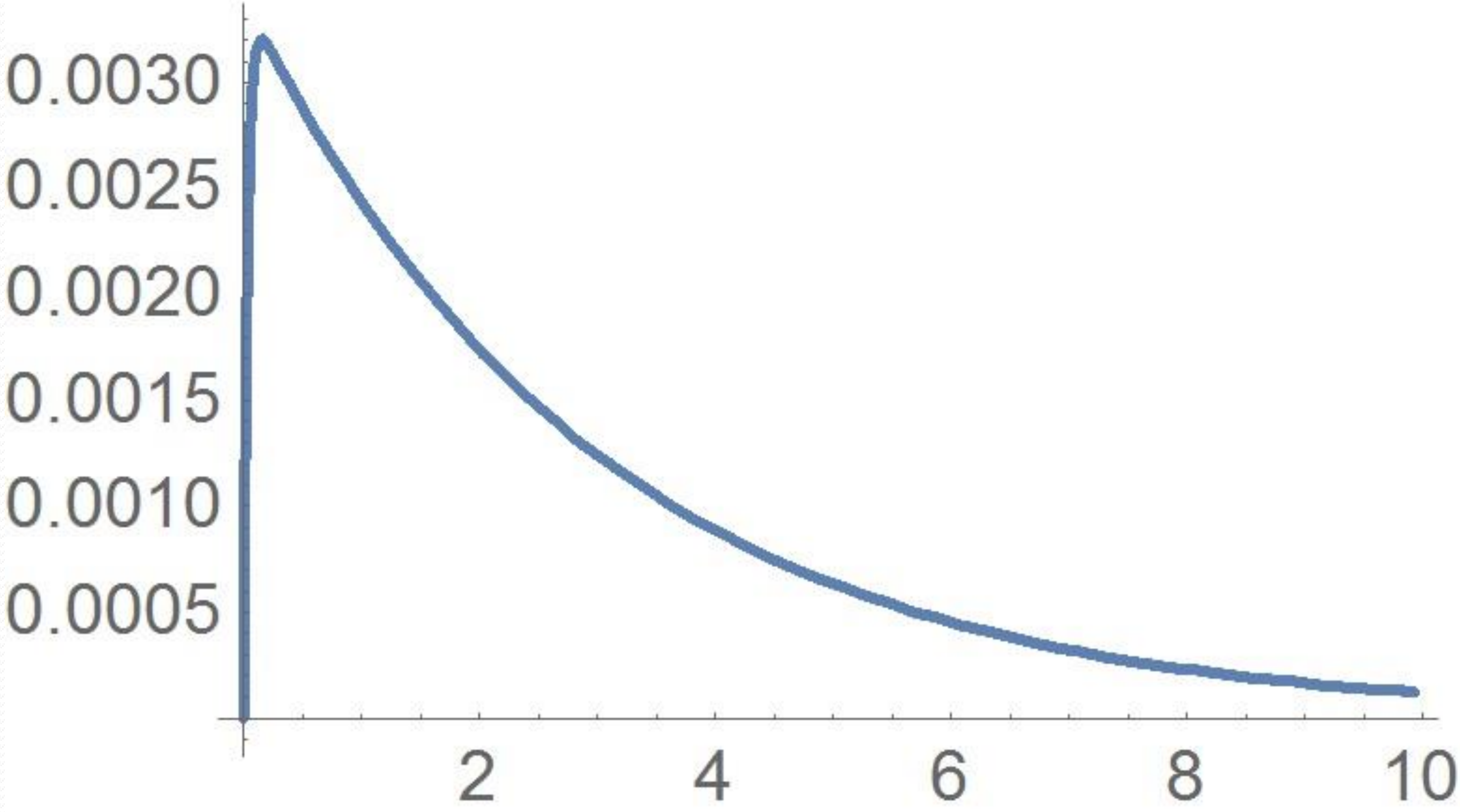
$$\lambda_{1,2} = -\frac{c}{2m} \pm \frac{1}{2}\sqrt{\frac{c^2}{m^2} - 4\frac{k}{m}}$$

$$\lambda_{1,2} = -\frac{c}{2m} \pm \frac{1}{2}\sqrt{\frac{c^2}{m^2} - 4\frac{k}{m}}$$

$$Si \ \frac{c^2}{m^2} - 4\frac{k}{m} > 0$$

$$\lambda_1 = Re1$$
$$\lambda_2 = Re2$$

$$y_h = C1\, e^{\lambda 1\, t} + C2\, e^{\lambda 2\, t}$$

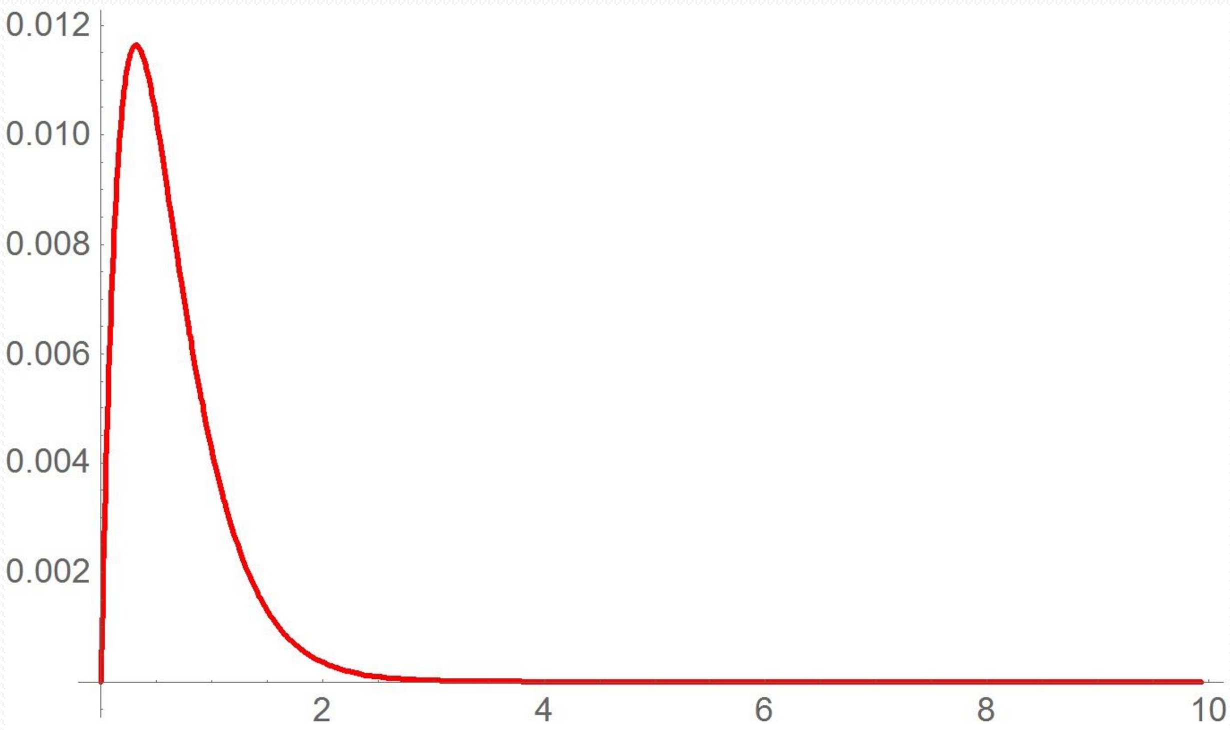

0.0030
0.0025
0.0020
0.0015
0.0010
0.0005
2
4
6
8
10

$$\lambda_{1,2} = -\frac{c}{2m} \pm \frac{1}{2}\sqrt{\frac{c^2}{m^2} - 4\frac{k}{m}}$$

$$Si\ \frac{c^2}{m^2} - 4\frac{k}{m} = 0$$

$$\lambda_1 = -\frac{c}{2m} = \lambda_2$$

$$y_h = C1\, e^{\lambda\, t} + t\, C2\, e^{\lambda\, t}$$

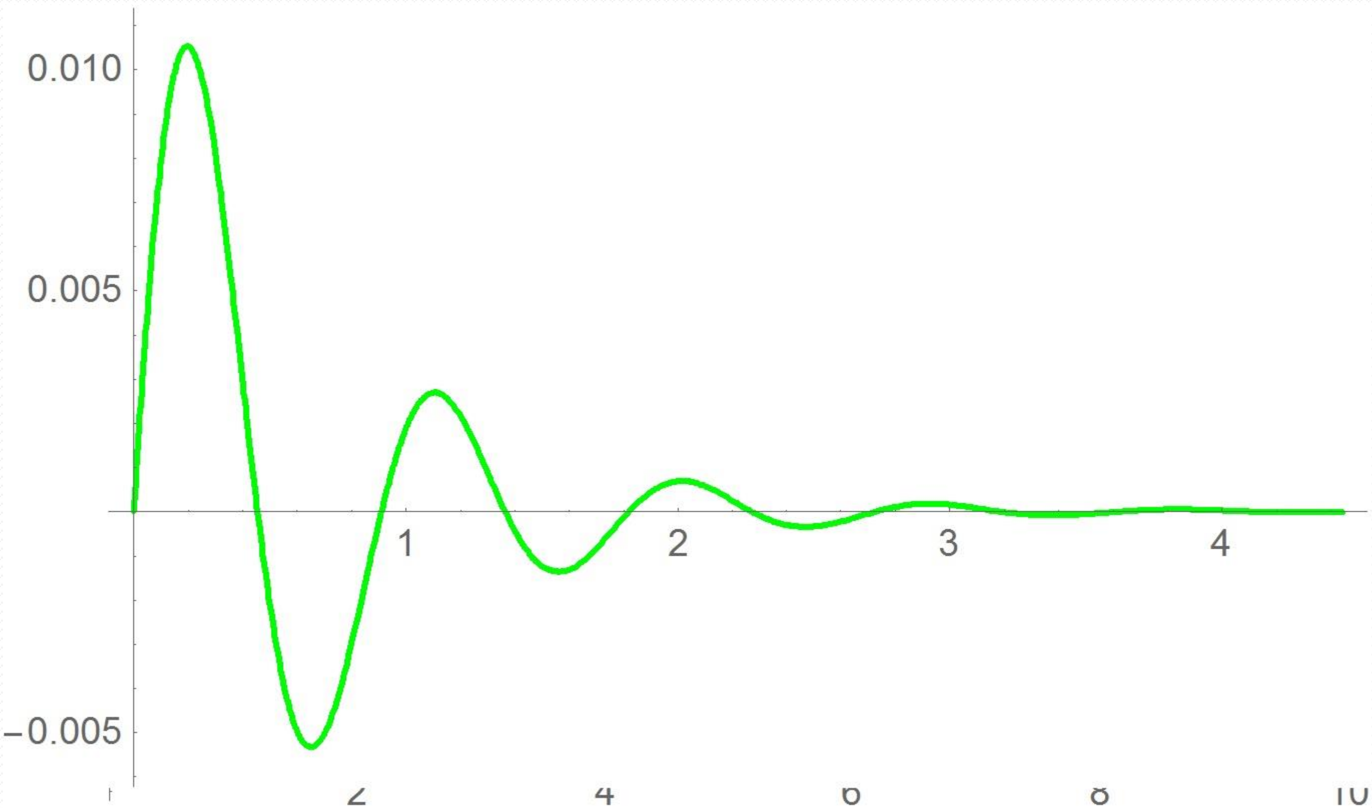

0.012
0.010
0.008
0.006
0.004
0.002
2
4
6
8
10

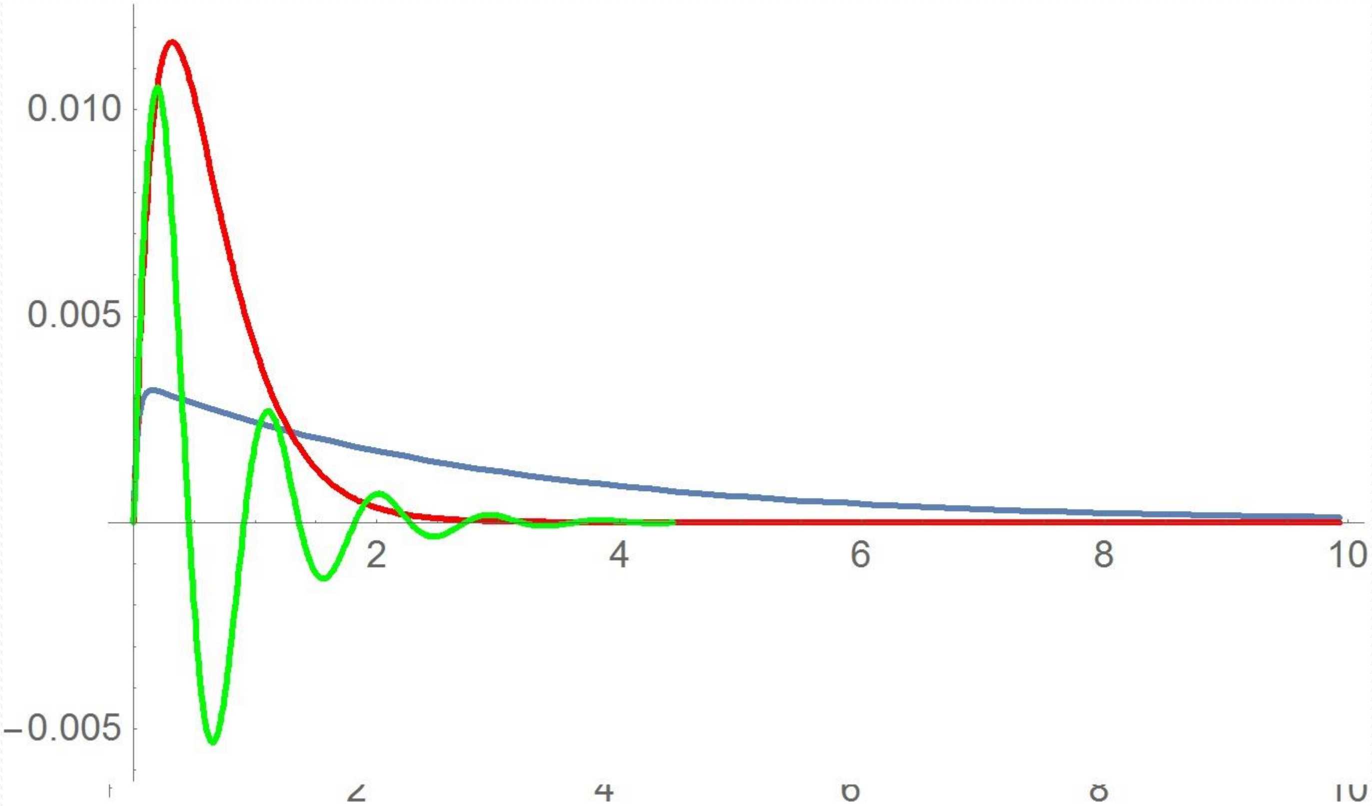
$$\lambda_{1,2} = -\frac{c}{2m} \pm \frac{1}{2}\sqrt{\frac{c^2}{m^2} - 4\frac{k}{m}}$$

$$Si \ \frac{c^2}{m^2} - 4\frac{k}{m} < 0$$

$$\lambda_{1,2} = \alpha \pm \beta\, i$$

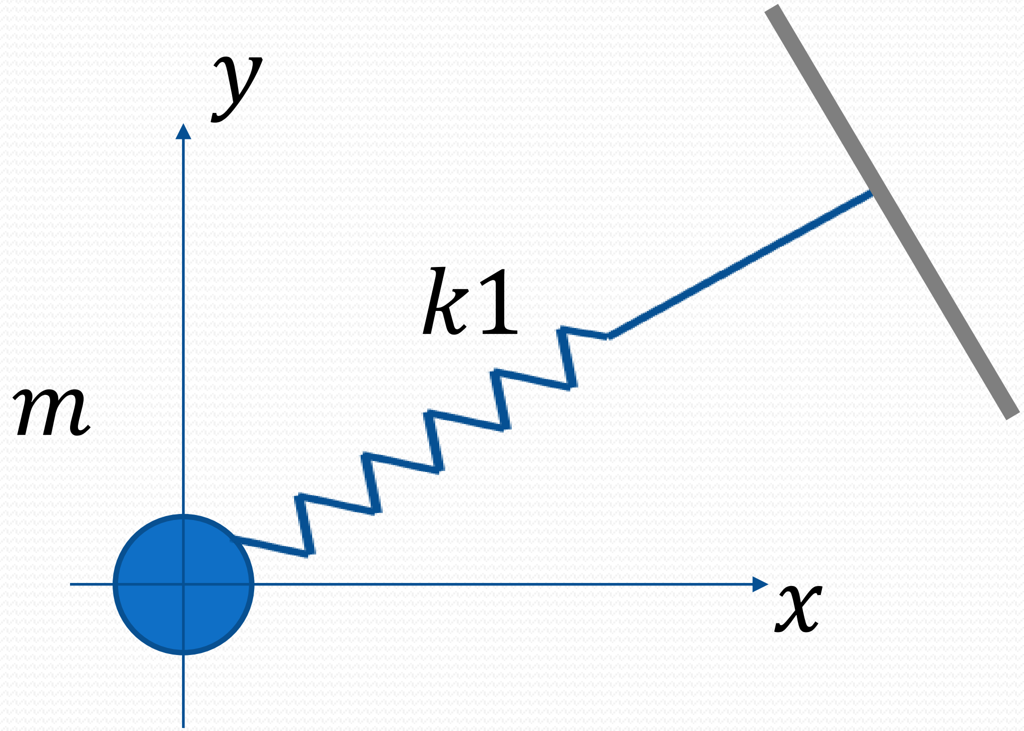
$$y_h = C1\, e^{\alpha\, t} Cos(\beta\, t) + C2\, e^{\alpha\, t} Sin(\beta\, t)$$


0.010
0.005
−0.005
1
2
3
4
2
4
6
8
10

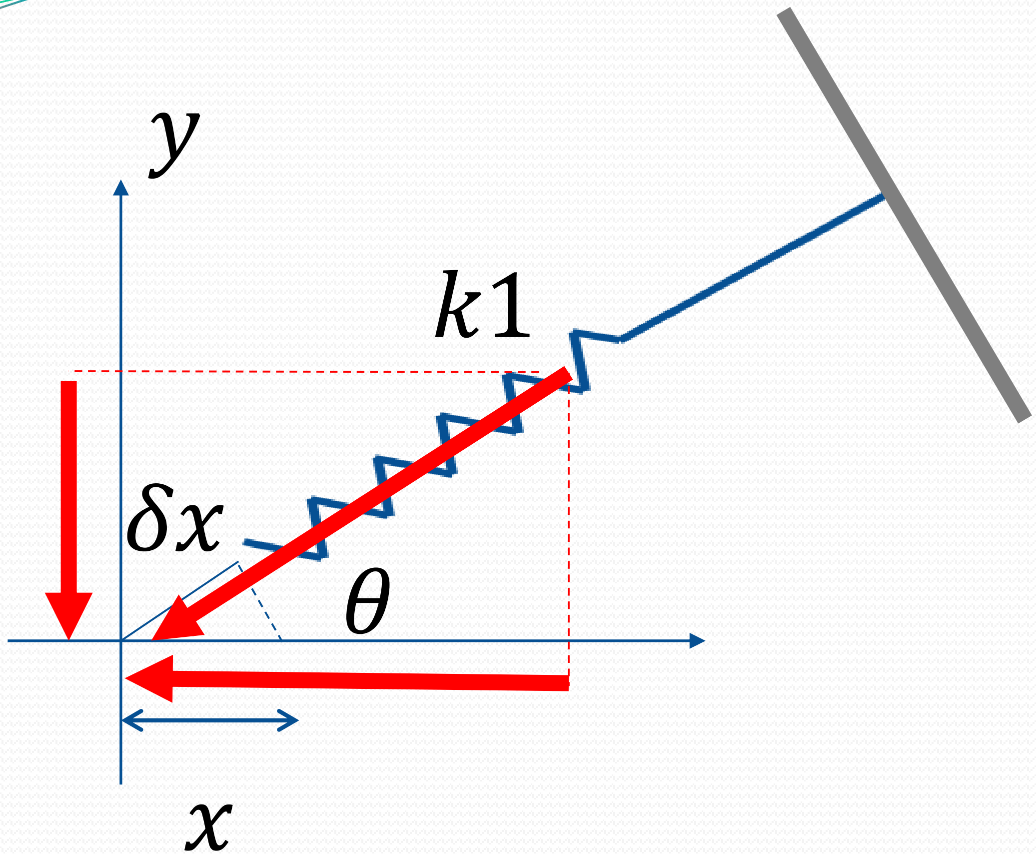

0.010
0.005
−0.005
2
4
6
8
10

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

Modelado con dos grados de libertad.



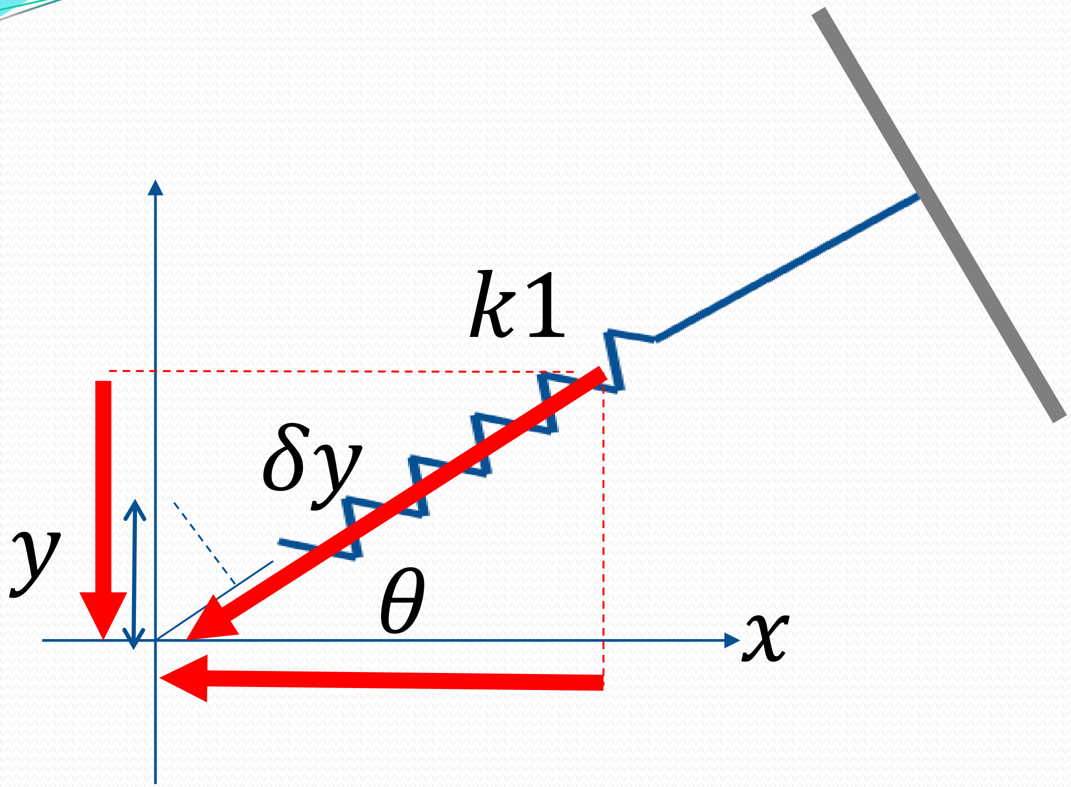
y
$k1$
m
x



$$\delta_x = x \, Cos \, \theta$$

$$Fxx = -k1 \, x \, Cos \, \theta \, Cos \, \theta$$

$$Fyx = -k1 \, x \, Cos \, \theta \, Sen \, \theta$$



$$\delta_y = y\,Sen\,\theta$$

$$Fxy = -k1\,y\,Sen\,\theta\,Cos\,\theta$$

$$Fyy = -k1\,y\,Sen\,\theta\,Sen\,\theta$$

$$Fxx = -k1\, x\, Cos\, \theta\, Cos\, \theta$$

$$Fyx = -k1\, x\, Cos\, \theta\, Sen\, \theta$$

$$Fxy = -k1\, y\, Sen\, \theta\, Cos\, \theta$$

$$Fyy = -k1\, y\, Sen\, \theta\, Sen\, \theta$$

$$\sum Fx = -Fxx - Fxy$$

$$\sum Fy = -Fyx - Fyy$$

$$\sum Fx = -Fxx - Fxy$$

$$\sum Fy = -Fyx - Fyy$$

$$\sum Fx = -k1\, x\, Cos\, \theta\, Cos\, \theta - k1\, y\, Sen\, \theta\, Cos\, \theta$$

$$\sum Fy = -k1\, x\, Cos\, \theta\, Sen\, \theta - k1\, y\, Sen\, \theta\, Sen\, \theta$$

$$\sum Fx = -k1\, x\, Cos\, \theta\, Cos\, \theta - k1\, y\, Sen\, \theta\, Cos\, \theta$$

$$\sum Fy = -k1\, x\, Cos\, \theta\, Sen\, \theta - k1\, y\, Sen\, \theta\, Sen\, \theta$$

$$kxx = k1\, Cos\, \theta\, Cos\, \theta$$

$$kyy = k1\, Sen\, \theta\, Sen\, \theta$$

$$kxy = k1\, Cos\, \theta\, Sen\, \theta$$

$$\sum Fx = -k1\, x\, Cos\,\theta\, Cos\,\theta - k1\, y\, Sen\,\theta\, Cos\,\theta$$

$$\sum Fy = -k1\, x\, Cos\,\theta\, Sen\,\theta - k1\, y\, Sen\,\theta\, Sen\,\theta$$

$$\sum Fx = -x\, kxx - y\, kxy$$

$$\sum Fy = -x\, kxy - y\, kyy$$

$$\sum Fx = -x\,kxx - y\,kxy$$

$$\sum Fy = -x\,kxy - y\,kyy$$

$$-x\,kxx - y\,kxy = m\,\ddot{x}$$

$$-x\,kxy - y\,kyy = m\,\ddot{y}$$

$$-x\,kxx\;-y\,kxy=m\,\ddot{x}$$

$$-x\,kxy-y\,kyy=m\,\ddot{y}$$

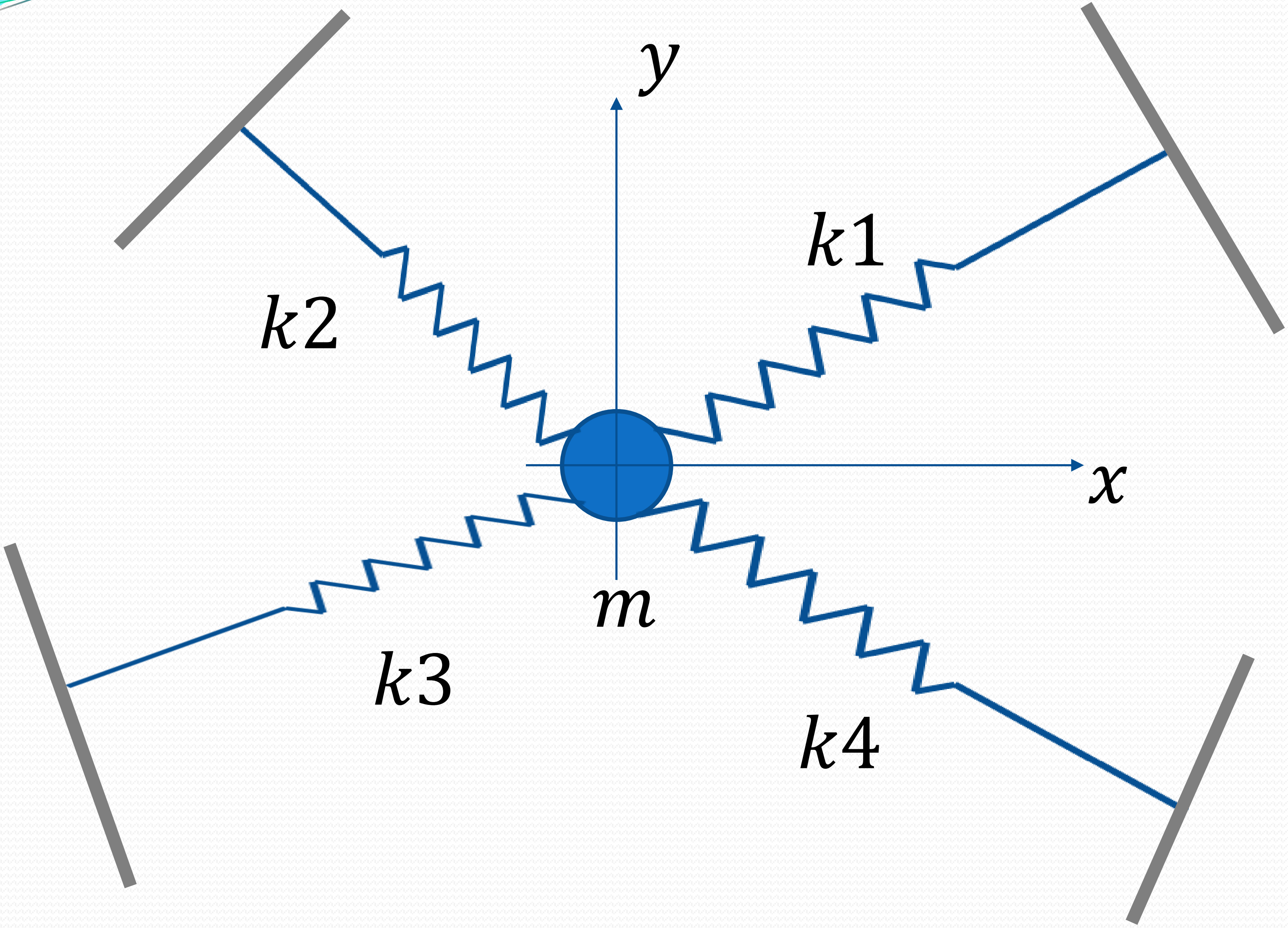
$$m\,\ddot{x}+x\,kxx+y\,kxy=0$$

$$m\,\ddot{y}+x\,kxy+y\,kyy=0$$

$$m\,\ddot{x} + x\,kxx + y\,kxy = 0$$

$$m\,\ddot{y} + x\,kxy + y\,kyy = 0$$

$$\begin{bmatrix} D^2 + kxx & kxy \\ kxy & D^2 + kyy \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



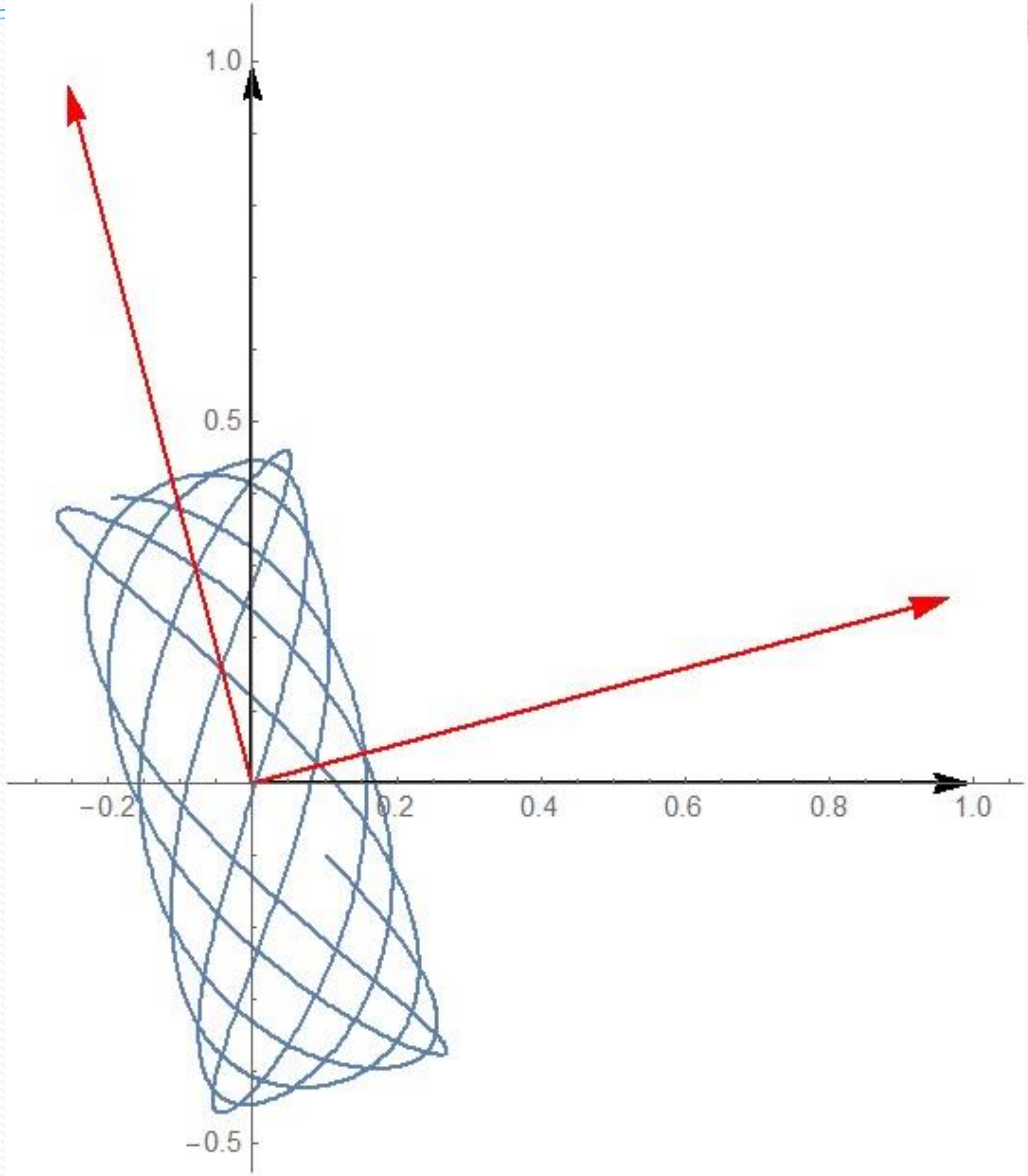
y
$k1$
$k2$
x
m
$k3$
$k4$

$$kxx = \sum k_i \,(Cos\ \theta_i)^2$$

$$kyy = \sum k_i \,(Sen\ \theta_i)^2$$

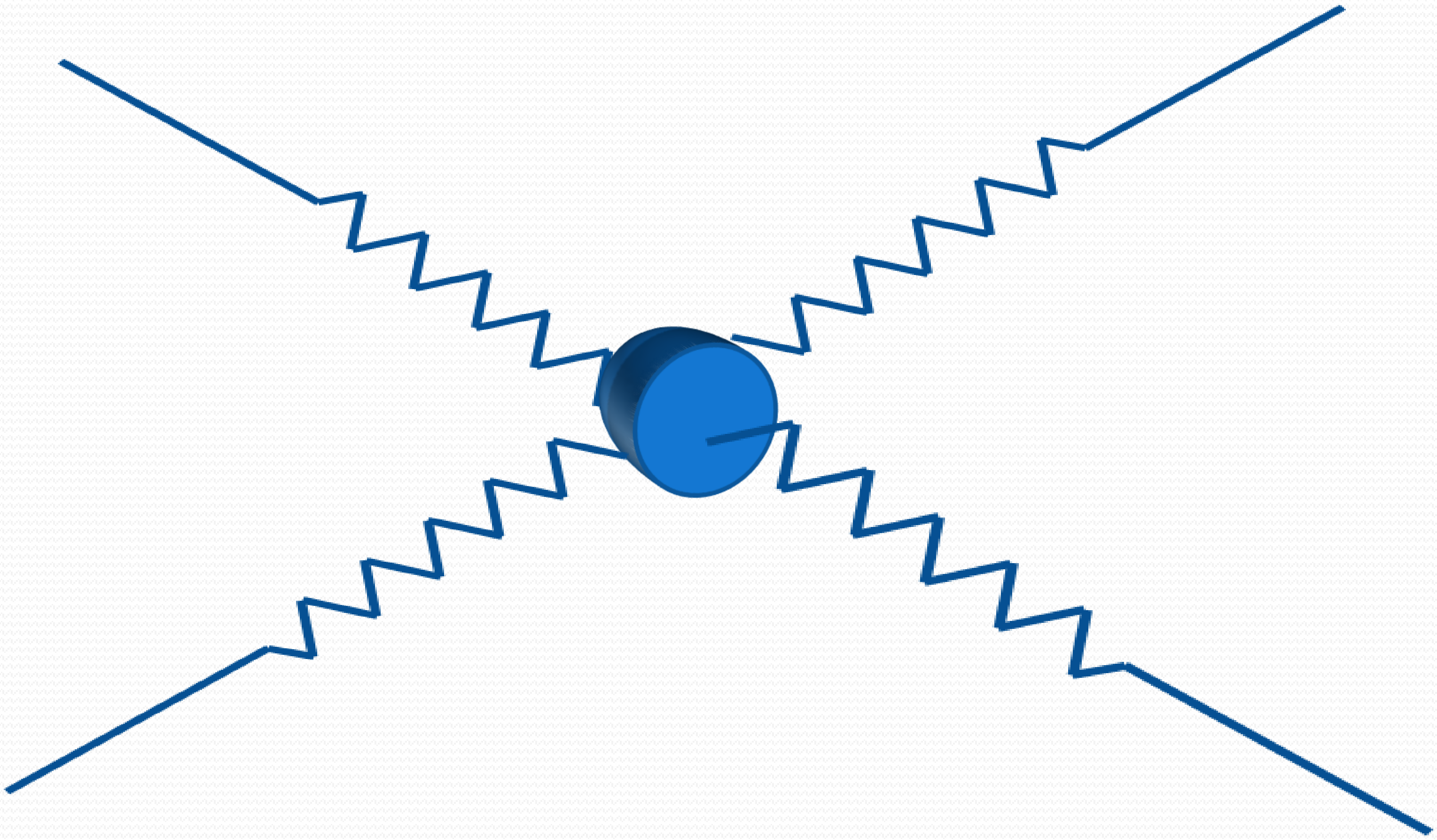
$$kxy = \sum k_i \, Sen\ \theta_i \, Cos\ \theta_i$$

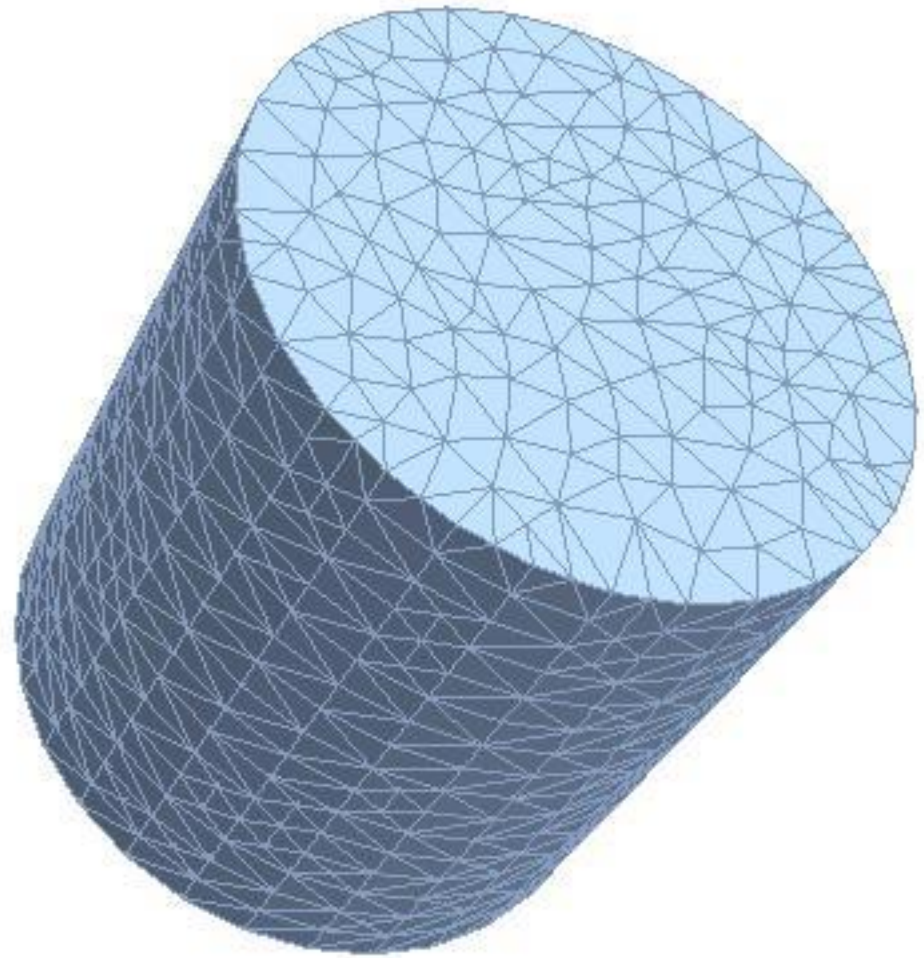
$$\begin{bmatrix} D^2 + kxx & kxy \\ kxy & D^2 + kyy \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



1.0
0.5
−0.5
−0.2
0.2
0.4
0.6
0.8
1.0

Sistemas con múltiples grados de libertad.





Vibración amortiguada forzada.

Función de transferencia

In[52]:=
```
Clear[Gs, m, k, c, ωn, ξ]
```
$$Gs = \frac{1}{m\left(s^2 + 2\,\xi\,\omega n\,s + \omega n^2\right)}$$

Out[53]= $$\frac{1}{m\left(s^2 + 2\,s\,\xi\,\omega n + \omega n^2\right)}$$

Respuesta a un impulso

Si $\zeta > 1$

In[54]:=
```
Clear[m, k, c, Fs, Xs]
```
$$Fs = e^{-s\,t0}$$
$$Xs = Gs\,Fs$$

Out[55]= 1

Out[56]= $$\frac{1}{m\left(s^2 + 2\,s\,\xi\,\omega n + \omega n^2\right)}$$

In[57]:= $$\omega n = \sqrt{\frac{k}{m}}$$
$$\xi = \frac{c}{\sqrt{4\,m\,k}}$$
$$\tau n = \frac{2\,\pi}{\omega n}$$

Out[57]= $$\sqrt{\frac{k}{m}}$$

Out[58]= $$\frac{c}{2\sqrt{k\,m}}$$

Out[59]= $$\frac{2\,\pi}{\sqrt{\frac{k}{m}}}$$

In[60]:=
```
Apart[Gs]
```

Out[60]=
$$\frac{1}{2\left(k-c\,s+m\,s^2\right)}-\frac{\sqrt{\frac{k}{m}}\,\sqrt{k\,m}}{2\,k\left(k-c\,s+m\,s^2\right)}+\frac{1}{2\left(k+c\,s+m\,s^2\right)}+\frac{\sqrt{\frac{k}{m}}\,\sqrt{k\,m}}{2\,k\left(k+c\,s+m\,s^2\right)}$$

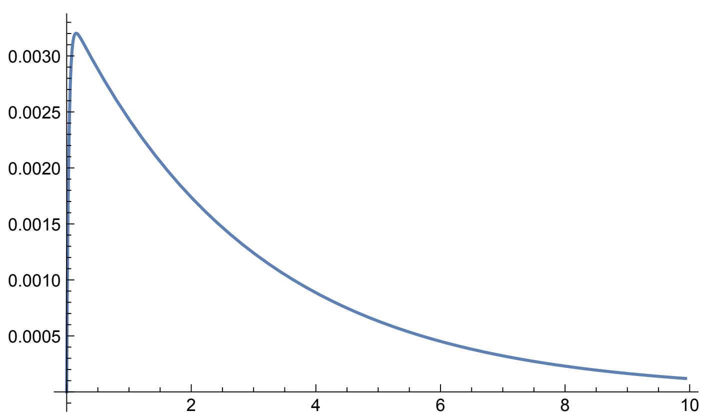
In[61]:=
```
m = 10;
k = 100;
c = 300;
t0 = 0;
```

In[65]:=
```
ξ // N
Xs
xt = InverseLaplaceTransform[Xs, s, t]
Grafica1 = Plot[xt, {t, 0, 5 τn}, PlotRange → All]
```

Out[65]= 4.74342

Out[66]=
$$\frac{1}{10\left(10+30\,s+s^2\right)}$$

Out[67]=
$$-\frac{e^{\left(-15-\sqrt{215}\right)t}-e^{\left(-15+\sqrt{215}\right)t}}{20\sqrt{215}}$$

Out[68]=


Si $\zeta = 1$

In[69]:=
```
Clear[m, k, c, Fs, Xs]
Fs = ⅇ^(-s t0)
Xs = Gs Fs
```

Out[70]= 1

Out[71]= $$\frac{1}{m\left(\frac{k}{m}+\frac{c\sqrt{\frac{k}{m}}\,s}{\sqrt{k\,m}}+s^2\right)}$$

In[72]:=
$$\omega n = \sqrt{\frac{k}{m}}$$
$$\xi = \frac{c}{\sqrt{4\,m\,k}}$$
$$\tau n = \frac{2\,\pi}{\omega n}$$

Out[72]= $\sqrt{\frac{k}{m}}$

Out[73]= $\frac{c}{2\sqrt{k\,m}}$

Out[74]= $\frac{2\,\pi}{\sqrt{\frac{k}{m}}}$

In[75]:=
```
Apart[Gs]
```

Out[75]= $$\frac{1}{2\,(k-c\,s+m\,s^2)}-\frac{\sqrt{\frac{k}{m}}\,\sqrt{k\,m}}{2\,k\,(k-c\,s+m\,s^2)}+\frac{1}{2\,(k+c\,s+m\,s^2)}+\frac{\sqrt{\frac{k}{m}}\,\sqrt{k\,m}}{2\,k\,(k+c\,s+m\,s^2)}$$

In[76]:=
```
m = 10;
k = 100;
c = 63.2456;
t0 = 0;
ξ // N
```

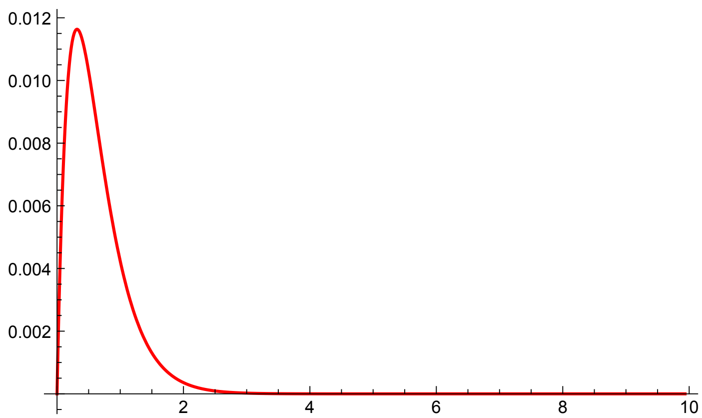
Out[80]= 1.

In[81]:=
```
Xs
xt = InverseLaplaceTransform[Xs, s, t]
Grafica2 = Plot[xt, {t, 0, 5 τn}, PlotRange → All, PlotStyle → Red]
```

Out[81]= $\frac{1}{10\left(10+6.32456\, s+s^{2}\right)}$

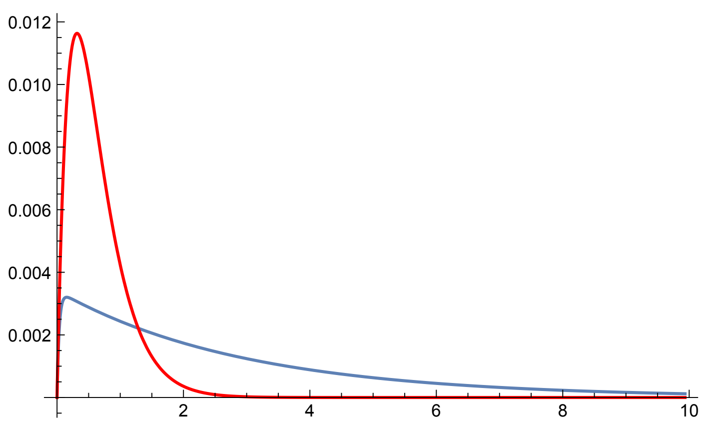
Out[82]= $\frac{1}{10}\left(-129.976\, e^{-3.16613\, t}+129.976\, e^{-3.15843\, t}\right)$

Out[83]=



In[84]:=
```
Show[{Grafica1, Grafica2}]
```

Out[84]=



Si $\zeta < 1$

In[85]:=
```
Clear[m, k, c, Fs, Xs]
Fs = e^(-s t0)
Xs = Gs Fs
```

Out[86]= 1

Out[87]= $$\frac{1}{m\left(\frac{k}{m}+\frac{c\sqrt{\frac{k}{m}}\,s}{\sqrt{k\,m}}+s^2\right)}$$

In[88]:=
$$\omega n = \sqrt{\frac{k}{m}}$$
$$\xi = \frac{c}{\sqrt{4\,m\,k}}$$
$$\tau n = \frac{2\,\pi}{\omega n}$$

Out[88]= $\sqrt{\frac{k}{m}}$

Out[89]= $\frac{c}{2\sqrt{k\,m}}$

Out[90]= $\frac{2\,\pi}{\sqrt{\frac{k}{m}}}$

In[91]:=
```
Apart[Gs]
```

Out[91]= $$\frac{1}{2\left(k-c\,s+m\,s^2\right)}-\frac{\sqrt{\frac{k}{m}}\sqrt{k\,m}}{2\,k\left(k-c\,s+m\,s^2\right)}+\frac{1}{2\left(k+c\,s+m\,s^2\right)}+\frac{\sqrt{\frac{k}{m}}\sqrt{k\,m}}{2\,k\left(k+c\,s+m\,s^2\right)}$$

In[92]:=
```
m = 10;
k = 500;
c = 30;
t0 = 0;
ξ // N
```

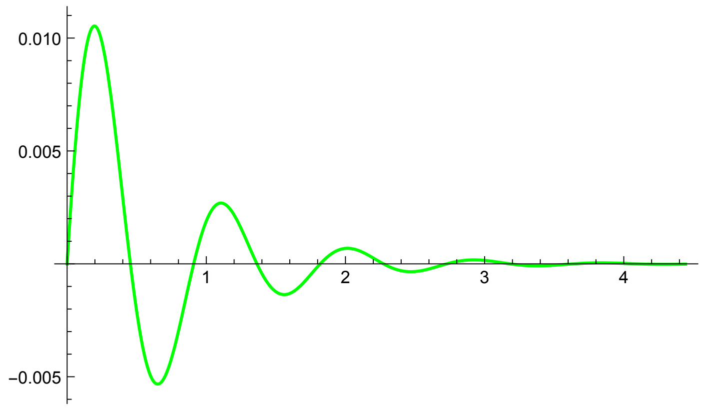
Out[96]= 0.212132

In[97]:=
```
Xs
xt = InverseLaplaceTransform[Xs, s, t]
Grafica3 = Plot[xt, {t, 0, 5 τn}, PlotRange → All, PlotStyle → Green]
```

Out[97]= $$\frac{1}{10\left(50+3\,s+s^2\right)}$$

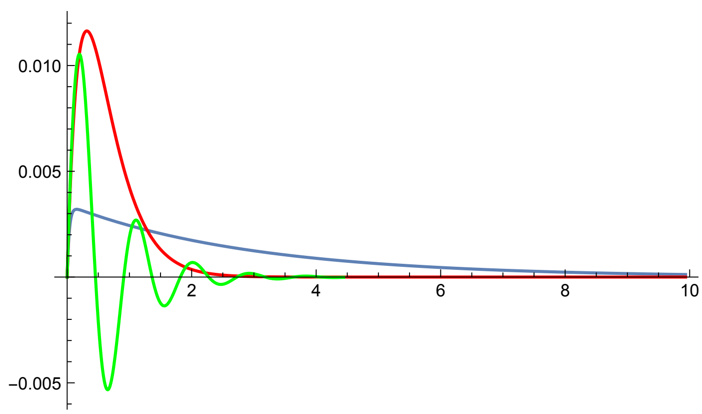
Out[98]= $$\frac{e^{-3\,t/2}\,\mathrm{Sin}\left[\frac{\sqrt{191}\,t}{2}\right]}{5\,\sqrt{191}}$$

Out[99]=



In[100]:=
```
Show[Grafica1, Grafica2, Grafica3]
```

Out[100]=



In[101]:= **Manipulate**[

$$\mathbf{Gs} = \frac{1}{\mathbf{m}\left(\mathbf{s}^2 + 2\,\xi\,\omega\mathbf{n}\,\mathbf{s} + \omega\mathbf{n}^2\right)};$$

$$\omega\mathbf{n} = \sqrt{\frac{\mathbf{k}}{\mathbf{m}}};$$

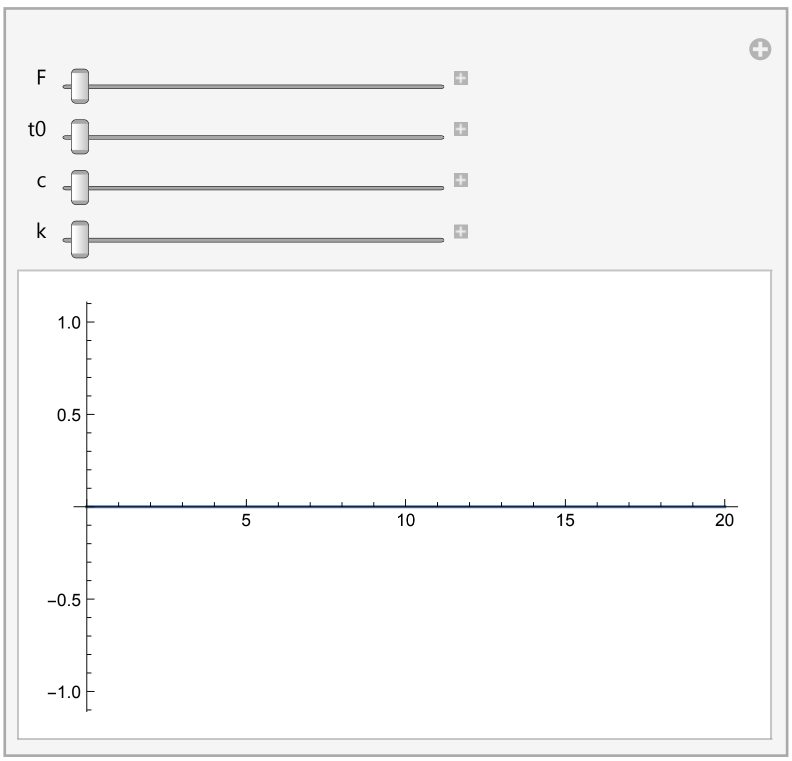
$$\xi = \frac{\mathbf{c}}{\sqrt{4\,\mathbf{m}\,\mathbf{k}}};$$

$$\tau\mathbf{n} = \frac{2\,\pi}{\omega\mathbf{n}};$$

$$\mathbf{Fs} = \mathbf{F}\,e^{-\mathbf{s}\,\mathbf{t0}};$$

```
Xs = Gs Fs;
xt = InverseLaplaceTransform[Xs, s, t];
Plot[Evaluate[xt], {t, 0, 20}, PlotRange → All]
, {F, 0, 100}, {t0, 0, 20}, {c, 0, 100}, {k, 1, 1000}]
```

Out[101]=



In[102]:= **Manipulate[**

$$Gs = \frac{1}{m\left(s^2 + 2\,\zeta\,\omega n\,s + \omega n^2\right)};$$

$$\omega n = \sqrt{\frac{k}{m}};$$

$$\zeta = \frac{c}{\sqrt{4\,m\,k}};$$

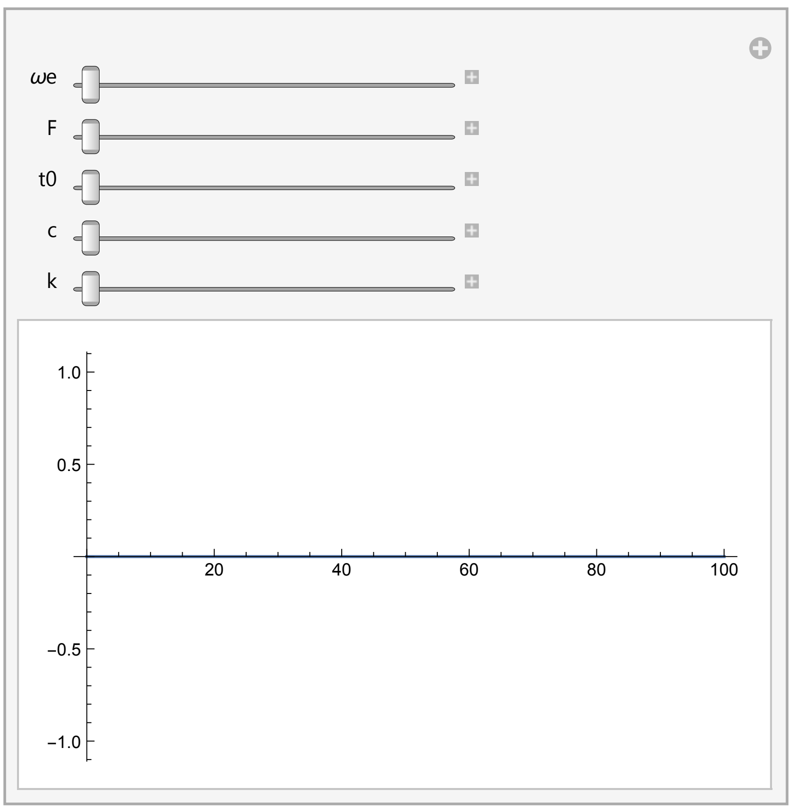
$$\tau n = \frac{2\,\pi}{\omega n};$$

$$Fs = F\,\frac{\omega e\,e^{-s\,t0}}{s^2 + \omega e^2};$$

```
Xs = Gs Fs;
xt = InverseLaplaceTransform[Xs, s, t];
Plot[Evaluate[xt], {t, 0, 100}, PlotRange → All]

, {ωe, 0, 4}, {F, 0, 100}, {t0, 0, 20}, {c, 0, 100}, {k, 1, 1000}]
```

Out[102]=



In[3]:=

```
Clear[kxx, kxy, kyy, matK, matM, m, k1, θ1, k2, θ2, k3, θ3, k4, θ4]
```

Datos

In[4]:=

```
k1 = 100;
θ1 = 45 °;
k2 = 10;
θ2 = 120 °;
k3 = 100;
θ3 = 200 °;
k4 = 100;
θ4 = -31 °;
m = 10;
```

In[13]:=

```
matM = {{m, 0}, {0, m}};
matM // MatrixForm
```

Out[14]//MatrixForm=

$$\begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$$

In[15]:=

```
Inverse[matM] // MatrixForm
```

Out[15]//MatrixForm=

$$\begin{pmatrix} \frac{1}{10} & 0 \\ 0 & \frac{1}{10} \end{pmatrix}$$

In[16]:=

```
matK = {{kxx, kxy}, {kxy, kyy}};
matK // MatrixForm
```

Out[17]//MatrixForm=

$$\begin{pmatrix} kxx & kxy \\ kxy & kyy \end{pmatrix}$$

In[18]:=

```
matDin = Inverse[matM].matK
matDin // MatrixForm
```

Out[18]= $\left\{\left\{\frac{kxx}{10}, \frac{kxy}{10}\right\}, \left\{\frac{kxy}{10}, \frac{kyy}{10}\right\}\right\}$

Out[19]//MatrixForm=

$$\begin{pmatrix} \frac{kxx}{10} & \frac{kxy}{10} \\ \frac{kxy}{10} & \frac{kyy}{10} \end{pmatrix}$$

In[20]:=
```
kxx = k1 (Cos[θ1])^2 + k2 (Cos[θ2])^2 + k3 (Cos[θ3])^2 + k4 (Cos[θ4])^2
kyy = k1 (Sin[θ1])^2 + k2 (Sin[θ2])^2 + k3 (Sin[θ3])^2 + k4 (Sin[θ4])^2
kxy = k1 Sin[θ1] Cos[θ1] + k2 Sin[θ2] Cos[θ2] + k3 Sin[θ3] Cos[θ3] + k4 Sin[θ4] Cos[θ4]
```

Out[20]= $\frac{105}{2} + 100\,\text{Cos}[20\,°]^2 + 100\,\text{Cos}[31\,°]^2$

Out[21]= $\frac{115}{2} + 100\,\text{Sin}[20\,°]^2 + 100\,\text{Sin}[31\,°]^2$

Out[22]= $50 - \frac{5\sqrt{3}}{2} + 100\,\text{Cos}[20\,°]\,\text{Sin}[20\,°] - 100\,\text{Cos}[31\,°]\,\text{Sin}[31\,°]$

In[23]:=
```
{Valores, Vectores} = Eigensystem[Inverse[matM].matK] // Simplify // N
```

Out[23]= {{22.3167, 8.6833}, {{3.78597, 1.}, {-0.264133, 1.}}}

In[24]:=
```
λ1 = Valores[[1]]
λ2 = Valores[[2]]
```

Out[24]= 22.3167

Out[25]= 8.6833

In[26]:=
```
ω1 = √λ1 // N
ω2 = √λ2 // N
```

Out[26]= 4.72406

Out[27]= 2.94674

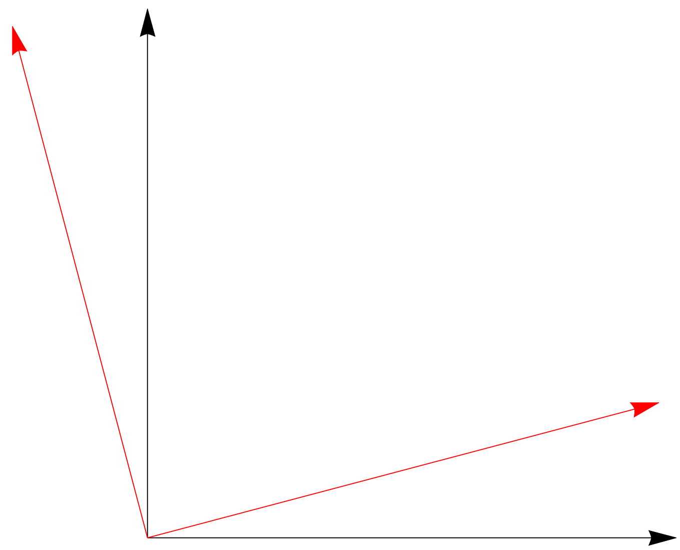
In[28]:=
```
Vec1 = Vectores[[1]] / Norm[Vectores[[1]]] // Simplify // N
Vec2 = Vectores[[2]] / Norm[Vectores[[2]]] // Simplify // N
```

Out[28]= {0.966842, 0.255375}

Out[29]= {-0.255375, 0.966842}

In[30]:=
```
Unitario1 = Graphics[{RGBColor[1, 0, 0], Arrow[{{0, 0}, Vec1}]}];
Unitario2 = Graphics[{RGBColor[1, 0, 0], Arrow[{{0, 0}, Vec2}]}];
vectI = Graphics[{RGBColor[0, 0, 0], Arrow[{{0, 0}, {1, 0}}]}];
vectJ = Graphics[{RGBColor[0, 0, 0], Arrow[{{0, 0}, {0, 1}}]}];
SistRef = {vectI, vectJ, Unitario1, Unitario2};
Show[SistRef]
```

Out[35]=



In[36]:=
```
Vec1.Vec2 // Simplify // Chop
```

Out[36]= 0

In[37]:=
```
C1 = Transpose[{Vec1, Vec2}]
```

Out[37]= {{0.966842, -0.255375}, {0.255375, 0.966842}}

In[38]:=
```
C1 // MatrixForm
matDiag = Inverse[C1].matDin.C1 // FullSimplify
matDiag // MatrixForm // Chop
```

Out[38]//MatrixForm=

$$\begin{pmatrix} 0.966842 & -0.255375 \\ 0.255375 & 0.966842 \end{pmatrix}$$

Out[39]= $\{\{22.3167, 0.\}, \{-8.88178\times 10^{-16}, 8.6833\}\}$

Out[40]//MatrixForm=

$$\begin{pmatrix} 22.3167 & 0 \\ 0 & 8.6833 \end{pmatrix}$$

In[41]:=
```
DiagonalMatrix[{λ1, λ2}] // N // Chop
```

Out[41]= {{22.3167, 0}, {0, 8.6833}}

In[42]:=
```
Chop[matDiag] == N[DiagonalMatrix[{λ1, λ2}]]
```

Out[42]= True

In[43]:=
```
{xsol, ysol} = A1 Vec1 Sin[ω1 t - ϕ1] + A2 Vec2 Sin[ω2 t - ϕ2]
```
Out[43]=
```
{0.966842 A1 Sin[4.72406 t - ϕ1] - 0.255375 A2 Sin[2.94674 t - ϕ2],
 0.255375 A1 Sin[4.72406 t - ϕ1] + 0.966842 A2 Sin[2.94674 t - ϕ2]}
```
In[44]:=
```
xsol // Simplify
ysol // Simplify
```
Out[44]=
```
0.966842 A1 Sin[4.72406 t - ϕ1] - 0.255375 A2 Sin[2.94674 t - ϕ2]
```
Out[45]=
```
0.255375 A1 Sin[4.72406 t - ϕ1] + 0.966842 A2 Sin[2.94674 t - ϕ2]
```
In[46]:=
```
matK // Simplify // MatrixForm
```
Out[46]//MatrixForm=

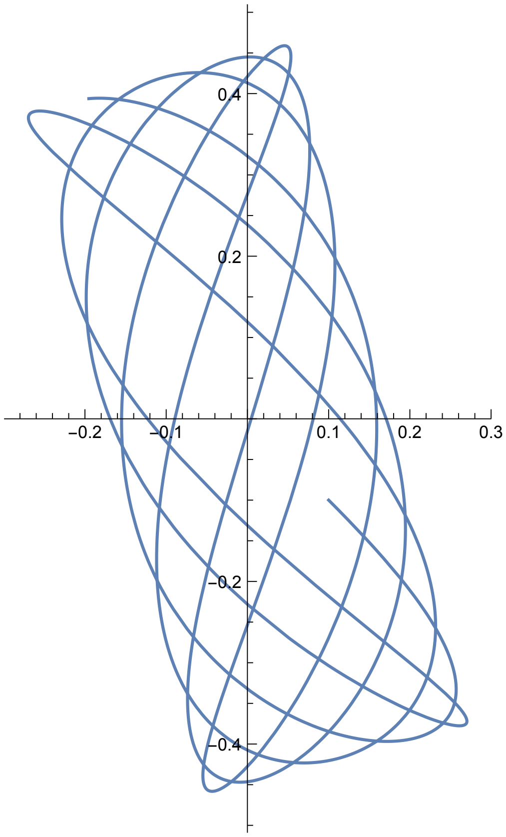
$$\begin{pmatrix} \frac{305}{2} + 50\,\mathrm{Cos}[40\,°] + 50\,\mathrm{Sin}[28\,°] & 50 - \frac{5\sqrt{3}}{2} - 50\,\mathrm{Cos}[28\,°] + 50\,\mathrm{Sin}[40\,°] \\ 50 - \frac{5\sqrt{3}}{2} - 50\,\mathrm{Cos}[28\,°] + 50\,\mathrm{Sin}[40\,°] & \frac{315}{2} - 50\,\mathrm{Cos}[40\,°] - 50\,\mathrm{Sin}[28\,°] \end{pmatrix}$$

In[47]:=
```
xsol
```
Out[47]=
```
0.966842 A1 Sin[4.72406 t - ϕ1] - 0.255375 A2 Sin[2.94674 t - ϕ2]
```
In[48]:=
```
xsol
ysol
```
Out[48]=
```
0.966842 A1 Sin[4.72406 t - ϕ1] - 0.255375 A2 Sin[2.94674 t - ϕ2]
```
Out[49]=
```
0.255375 A1 Sin[4.72406 t - ϕ1] + 0.966842 A2 Sin[2.94674 t - ϕ2]
```
In[50]:=
```
x0 = 0.1;
vx0 = 1;
y0 = -0.1;
vy0 = -1;
Sol1 = Solve[
  {xsol == x0, D[xsol, t] == vx0, ysol == y0, D[ysol, t] == vy0} /. t → 0, {A1, A2, ϕ1, ϕ2}]
{xSol1, ySol1} = {xsol, ysol} /. Sol1[[1]] // N
```
Solve::ifun :
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

Out[54]=
```
{{A1 → -0.166565, A2 → -0.432402, ϕ1 → 2.70026, ϕ2 → -0.286564},
 {A1 → 0.166565, A2 → -0.432402, ϕ1 → -0.441329, ϕ2 → -0.286564},
 {A1 → -0.166565, A2 → 0.432402, ϕ1 → 2.70026, ϕ2 → 2.85503},
 {A1 → 0.166565, A2 → 0.432402, ϕ1 → -0.441329, ϕ2 → 2.85503}}
```
Out[55]=
```
{0.161042 Sin[2.70026 - 4.72406 t] + 0.110425 Sin[0.286564 + 2.94674 t],
 0.0425364 Sin[2.70026 - 4.72406 t] - 0.418064 Sin[0.286564 + 2.94674 t]}
```

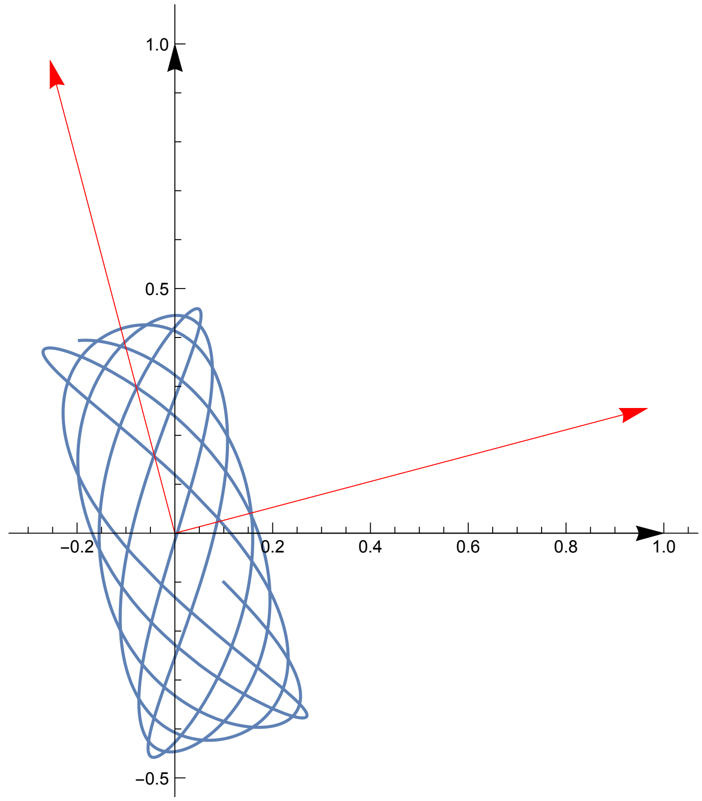
In[56]:=
```
Lisajous = ParametricPlot[{xSol1, ySol1}, {t, 0, 10}]
```

Out[56]=



In[57]:=
```
Show[Lisajous, SistRef, PlotRange → All]
```

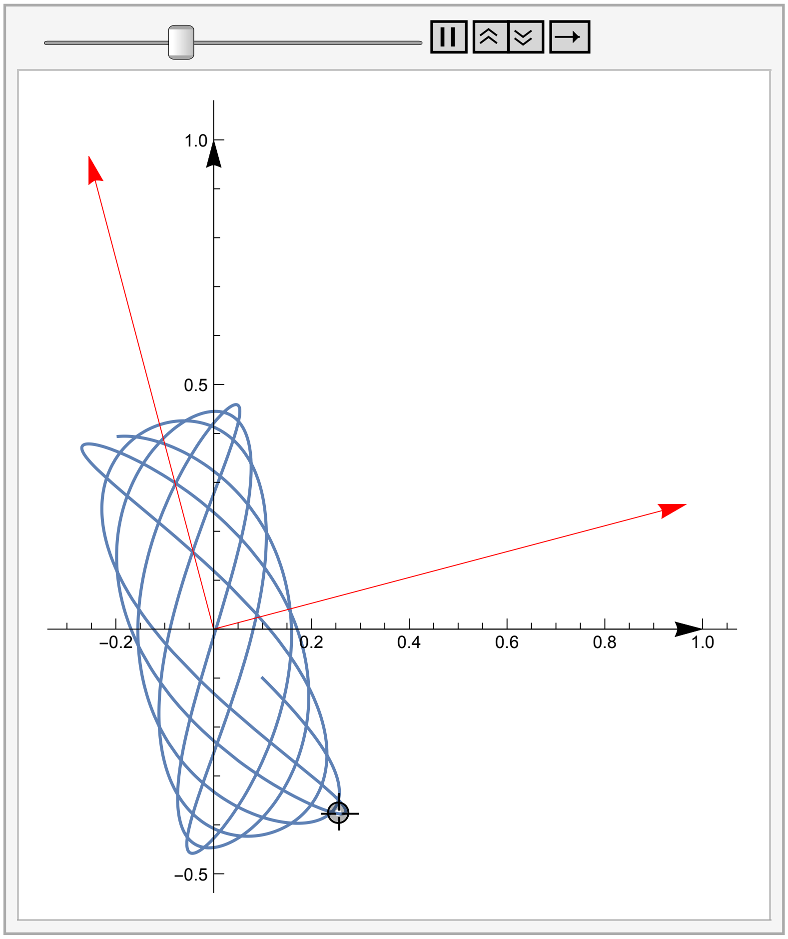
Out[57]=



Simulación

In[58]:=
```
PuntoTrabajo = Graphics[Locator[{xSol1, ySol1}]];
Table[Show[Lisajous, SistRef, PuntoTrabajo, PlotRange → All], {t, 0, 20, 0.1}];
ListAnimate[%]
```

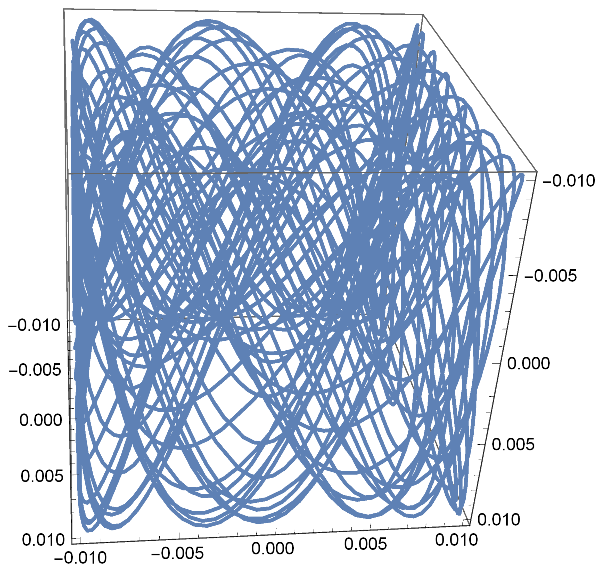
Out[60]=



Simulación 3D

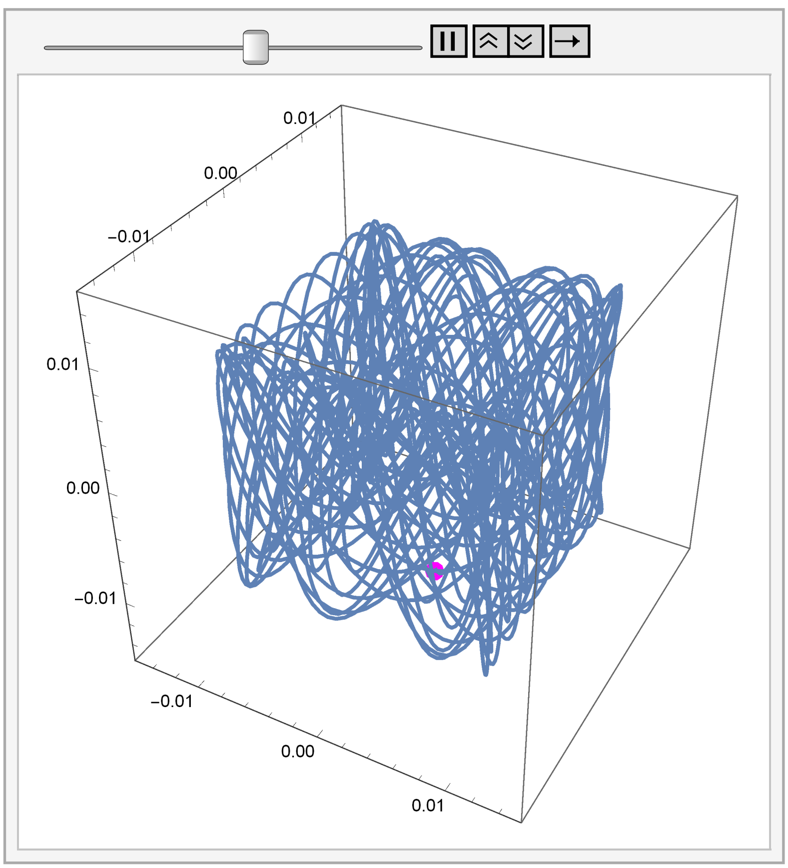
In[88]:=
```
Lisajous3D =
  ParametricPlot3D[{0.01 Sin[√5 t], 0.01 Sin[√7 t], 0.01 Sin[√27 t]}, {t, 0, 100}]
PuntoTrabajo2 = Graphics3D[{Magenta, AbsolutePointSize[10],
    Point[{0.01 Sin[√5 t], 0.01 Sin[√7 t], 0.01 Sin[√27 t]}]}];
Table[Show[Lisajous3D, PuntoTrabajo2, PlotRange →
    {{-0.015, 0.015}, {-0.015, 0.015}, {-0.015, 0.015}}], {t, 0, 25, 0.01}];
ListAnimate[%]
(*Export["Conferenca2.avi",%]*)
```

Out[88]=



−0.010
−0.005
0.000
0.005
0.010
−0.010
−0.005
0.000
0.005
0.010
−0.010
−0.005
0.000
0.005
0.010

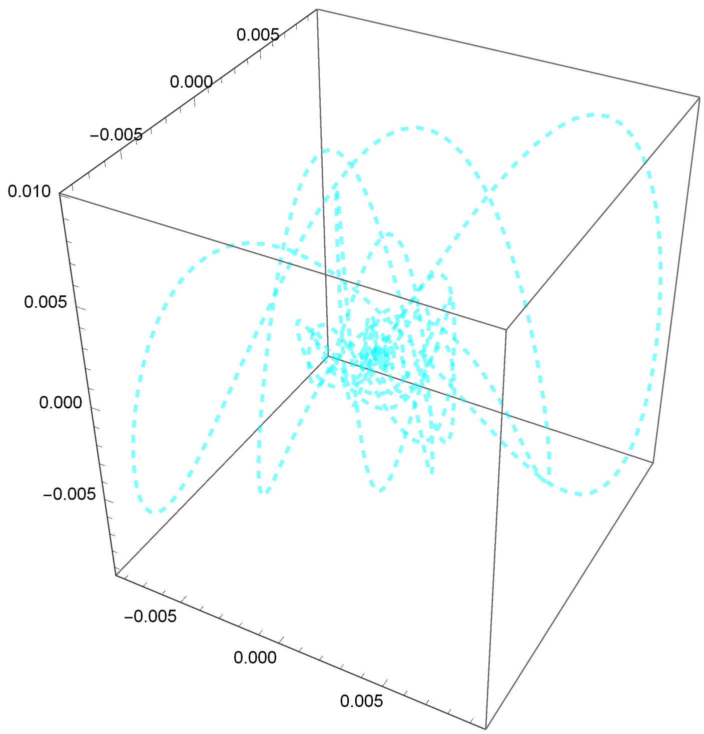
Out[91]=



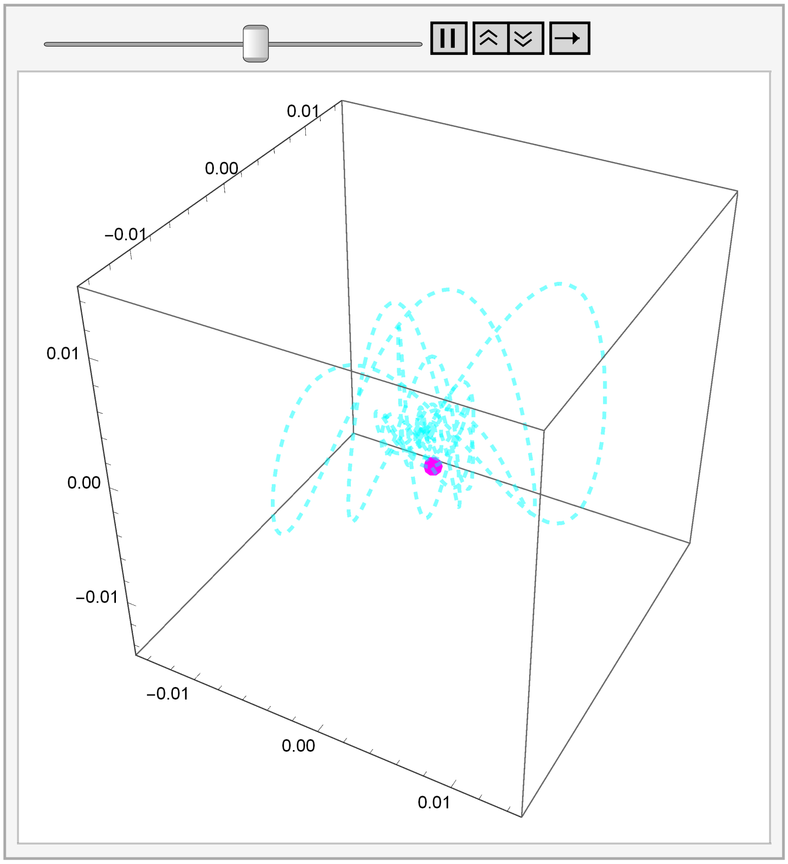
0.01
0.00
−0.01
0.01
0.00
−0.01
−0.01
0.00
0.01

In[96]:=
```
Lisajous3D =
  ParametricPlot3D[e^(-0.1 t) {0.01 Sin[√5 t], 0.01 Sin[√7 t], 0.01 Sin[√27 t]},
   {t, 0, 100}, PlotRange → All, PlotStyle → {Cyan, Dashed, Opacity[0.5]}]
PuntoTrabajo2 = Graphics3D[{Magenta, AbsolutePointSize[10],
    Point[e^(-0.1 t) {0.01 Sin[√5 t], 0.01 Sin[√7 t], 0.01 Sin[√27 t]}]}];
Table[Show[Lisajous3D, PuntoTrabajo2, PlotRange →
    {{-0.015, 0.015}, {-0.015, 0.015}, {-0.015, 0.015}}], {t, 0, 25, 0.1}];
ListAnimate[%]
(*Export["Conferenca3.avi",%]*)
```

Out[96]=

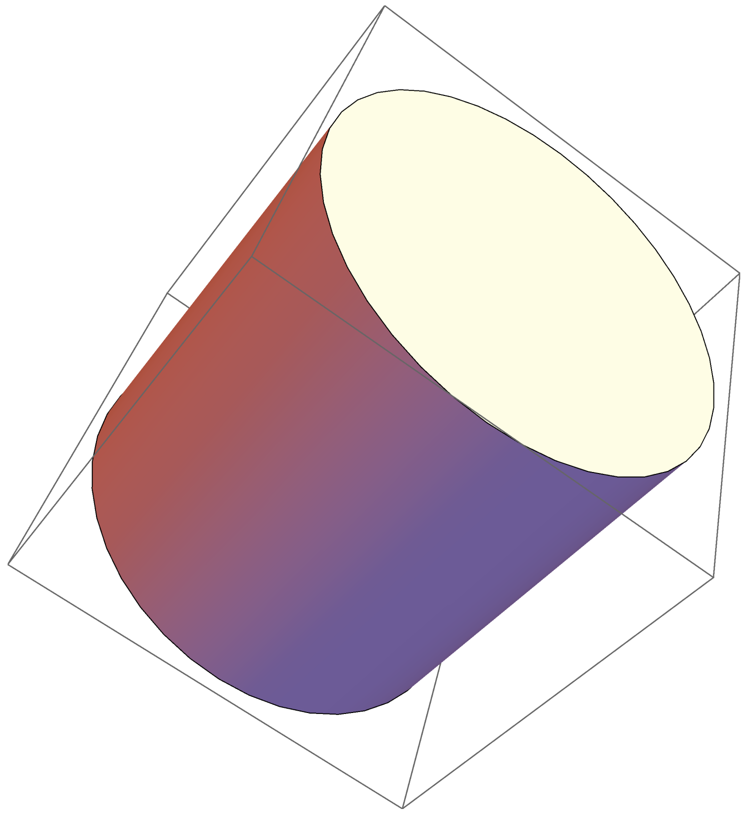


Out[99]=



In[65]:= `Show[Graphics3D[{Cylinder[]}]]`

Out[65]=



In[66]:=
```
DiscretizeGraphics[Cylinder[]]
```

Out[66]=